Case Study 2

Arjun, a mathematics enthusiast in Class 12, was preparing for his board exams. He came across a list of standard indefinite integrals and was fascinated by how these integrals could be verified using differentiation. His teacher explained that indefinite integration is essentially the reverse process of differentiation. For instance, since $\frac{d}{dx}(x^3) = 3x^2$, then $\int 3x^2 \, dx = x^3 + C$. Arjun started verifying other integrals like $\int e^x \, dx = e^x + C$ and $\int \frac{1}{x} \, dx = \ln|x| + C$ by differentiating the RHS. This gave him a stronger conceptual understanding and confidence. Let us now explore the key formulas and test the application of this concept.

Key Properties and Formulas:

- $\frac{d}{dx}[\int f(x) \, dx] = f(x)$

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

- $\int \frac{1}{x} \, dx = \ln|x| + C$

- $\int e^x \, dx = e^x + C$

- $\int a^x \, dx = \frac{a^x}{\ln a} + C$

MCQ Questions:

1. What is the value of $\int 5x^4 \, dx$?

    (a) $x^5 + C$

    (b) $5x^5 + C$

    (c) $\frac{5x^5}{5} + C$

    (d) $\frac{x^5}{5} + C$

    Answer: (c)
    Solution:
    $$\int 5x^4 \, dx = 5 \cdot \frac{x^5}{5} + C = x^5 + C$$

2. Which of the following is the correct verification for $\int e^x \, dx$?

    (a) $\frac{d}{dx}(e^x + C) = e^x$

    (b) $\frac{d}{dx}(e^x + C) = \ln e$

    (c) $\frac{d}{dx}(e^x + C) = x$

    (d) $\frac{d}{dx}(e^x + C) = 1$

    Answer: (a)
    Solution: The derivative of $e^x + C$ is e^x, which confirms the integral.

3. What is the value of $\int \frac{1}{x} \, dx$?

    (a) $\ln x + C$

    (b) $\ln|x| + C$

    (c) $x \ln x + C$

    (d) $\frac{1}{x} + C$

Answer: (b)
Solution:
$$\int \frac{1}{x}\,dx = \ln|x| + C$$

4. Which of the following is a correct pair of integral and its derivative?

   (a) $\int 2x\,dx = x^2 + C$, so $\frac{d}{dx}(x^2 + C) = 2x$

   (b) $\int 2x\,dx = 2x + C$, so $\frac{d}{dx}(2x + C) = 2$

   (c) $\int x\,dx = \frac{x^2}{2} + C$, so $\frac{d}{dx}(\frac{x^2}{2} + C) = x$

   (d) Both (a) and (c)

Answer: (d)
Solution: Both (a) and (c) show correct verification of the integral by differentiation.

5. What is the value of $\int 3e^x + \frac{2}{x}\,dx$?

   (a) $3e^x + 2x + C$

   (b) $3e^x + \ln|x| + C$

   (c) $3e^x + \frac{2}{x} + C$

   (d) $e^x + 2\ln x + C$

Answer: (b)
Solution:
$$\int (3e^x + \frac{2}{x})\,dx = 3e^x + 2\ln|x| + C$$

Option (b) is the closest correct choice. However, it lacks the coefficient 2 with $\ln|x|$. So, the correct answer should be:

Correct Answer: $3e^x + 2\ln|x| + C$ (Not listed exactly. Option (b) is partially correct but incomplete.)