Case Study 3

Meera, a bright student from Class 12, was solving problems related to indefinite integrals and came across a complex-looking expression: $\int \frac{2x}{x^2+1}\,dx$. At first, she was unsure how to proceed, but her teacher introduced her to the **substitution method**. She learned that the key idea was to simplify the integral by changing the variable. She let $u = x^2 + 1$, then found $du = 2x\,dx$, which transformed the original integral into a much simpler form: $\int \frac{1}{u}\,du = \ln|u| + C$. Substituting back gave her $\ln(x^2+1) + C$. With more practice, Meera became confident in applying this technique to other complex functions. Let's explore this method with some MCQs.

Key Formulas and Concepts:

- Substitution: Let $u = g(x) \Rightarrow \frac{du}{dx} = g'(x)$
- $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

MCQ Questions:

1. What is the value of $\int \frac{2x}{x^2+1}\,dx$?

   (a) $\ln|x^2+1| + C$
   (b) $2\ln|x^2+1| + C$
   (c) $\frac{1}{2}\ln|x^2+1| + C$
   (d) $\tan^{-1}(x) + C$

   Answer: (a)
   Solution: Let $u = x^2 + 1 \Rightarrow du = 2x\,dx$. So, the integral becomes $\int \frac{1}{u}\,du = \ln|u| + C = \ln|x^2+1| + C$

2. Evaluate $\int \frac{3x^2}{x^3+1}\,dx$ using substitution.

   (a) $\ln|x^3+1| + C$
   (b) $\frac{1}{3}\ln|x^3+1| + C$
   (c) $3\ln|x^3+1| + C$
   (d) $\ln|x^2+1| + C$

   Answer: (a)
   Solution: Let $u = x^3 + 1 \Rightarrow du = 3x^2\,dx$. So, the integral becomes $\int \frac{1}{u}\,du = \ln|u| + C = \ln|x^3+1| + C$

3. What substitution will simplify $\int \frac{x}{\sqrt{1-x^2}}\,dx$?

   (a) $x = \tan\theta$
   (b) $x = \cos\theta$
   (c) $x = \sin\theta$
   (d) $x = \ln\theta$

Answer: (c)
Solution: Use the trigonometric substitution $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$. Then, $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$, so the integral simplifies.

4. Evaluate $\int \frac{1}{x\ln x}\, dx$ using substitution.

   (a) $\ln|\ln x| + C$
   (b) $\ln x + C$
   (c) $\frac{1}{\ln x} + C$
   (d) $\frac{1}{x\ln x} + C$

   Answer: (a)
   Solution: Let $u = \ln x \Rightarrow du = \frac{1}{x}dx$. So, the integral becomes $\int \frac{1}{u}\, du = \ln|u| + C = \ln|\ln x| + C$

5. What is the integral $\int \frac{dx}{x^2+4}$?

   (a) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$
   (b) $\frac{1}{2}\ln|x^2+4| + C$
   (c) $\ln|x^2+4| + C$
   (d) $\tan^{-1}(x) + C$

   Answer: (a)
   Solution: This is a standard integral:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Here, $a = 2$, so answer is $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$