## Case Study 2: Formation of Differential Equations by Eliminating Arbitrary Constants

In engineering and physics, we often encounter families of curves defined by equations containing arbitrary constants. To describe the dynamics or behavior of such systems more precisely, we convert these families into differential equations by eliminating the arbitrary constants.

For example, the family of curves:

$$y = Ae^{2x} + Be^{-2x}$$

represents a general solution involving two arbitrary constants $A$ and $B$. By differentiating appropriately and eliminating these constants, we obtain a second-order differential equation representing this family:

$$\frac{d^2y}{dx^2} - 4y = 0$$

This is a process of converting a descriptive equation into a differential model. Understanding this helps students model physical systems such as harmonic oscillators, population growth, and electric circuits.

## MCQ Questions

1. The number of arbitrary constants in a family of curves equals the:

   (a) Degree of the differential equation

   (b) Order of the differential equation

   (c) Number of variables

   (d) Power of the highest derivative

   **Answer:** (b) Order of the differential equation

   **Solution:** The number of arbitrary constants in the general solution equals the order of the resulting differential equation after elimination.

2. The differential equation formed by eliminating constants from $y = Ae^{2x} + Be^{-2x}$ is:

   (a) $\frac{dy}{dx} + 2y = 0$

   (b) $\frac{d^2y}{dx^2} - 4y = 0$

   (c) $\frac{d^2y}{dx^2} + 4y = 0$

   (d) $\frac{dy}{dx} - 4y = 0$

   **Answer:** (b) $\frac{d^2y}{dx^2} - 4y = 0$

   **Solution:** Differentiating once: $y' = 2Ae^{2x} - 2Be^{-2x}$
   Differentiating again: $y'' = 4Ae^{2x} + 4Be^{-2x}$
   Using the original expression for $y$:

$$y'' = 4y \Rightarrow \frac{d^2y}{dx^2} - 4y = 0$$

3. Which method is used to form a differential equation from a family of curves?

   (a) Elimination of variables

   (b) Elimination of derivatives

   (c) Elimination of arbitrary constants

   (d) Substitution method

   **Answer:** (c) Elimination of arbitrary constants

   **Solution:** The process involves differentiating the equation and eliminating constants using the resulting expressions.

4. The order of the differential equation formed from $y = A\sin x + B\cos x$ is:

   (a) 1

   (b) 2

   (c) 3

   (d) 0

   **Answer:** (b) 2

   **Solution:** The expression contains two arbitrary constants, so the resulting differential equation is of order 2.

5. Which of the following is the correct differential equation formed from $y = A + Bx$?

   (a) $\frac{dy}{dx} = B$

   (b) $\frac{d^2y}{dx^2} = 0$

   (c) $\frac{dy}{dx} = A$

   (d) $\frac{dy}{dx} = x$

   **Answer:** (b) $\frac{d^2y}{dx^2} = 0$

   **Solution:** Differentiating once gives $y' = B$, differentiating again gives $y'' = 0$. Hence the differential equation is:

$$\frac{d^2y}{dx^2} = 0$$