Case Study 5: General and Particular Solutions of Differential Equations

In differential equations, the **general solution** contains an arbitrary constant (or constants), representing a family of curves. A **particular solution** is obtained by applying initial or boundary conditions to the general solution, determining the specific value of the constant(s).

For example, solving $\frac{dy}{dx} = 2x$ gives the general solution:

$$y = x^2 + C$$

If we are given an initial condition $y(1) = 5$, we substitute into the general solution to get:

$$5 = 1^2 + C \Rightarrow C = 4$$

So the particular solution is $y = x^2 + 4$.

This technique is useful in modeling real-world problems where initial values are known, such as population at a given time, temperature of an object at a specific moment, or displacement of a body at time zero.

MCQ Questions

1. The general solution of $\frac{dy}{dx} = 3x^2$ is:

   (a) $y = x^3 + 5$

   (b) $y = x^3 + C$

   (c) $y = 3x + C$

   (d) $y = x^2 + C$

   Answer: (b) $y = x^3 + C$

   Solution: Integrate both sides: $y = \int 3x^2 dx = x^3 + C$

2. If $y = x^3 + C$ and $y(2) = 10$, the particular solution is:

   (a) $y = x^3 + 2$

   (b) $y = x^3 + 3$

   (c) $y = x^3 - 2$

   (d) $y = x^3 + 5$

   Answer: (a) $y = x^3 + 2$

   Solution: Substitute $x = 2, y = 10 \Rightarrow 10 = 8 + C \Rightarrow C = 2$. So, the solution is $y = x^3 + 2$.

3. Which of the following is a general solution?

   (a) $y = x^2 + 7$

(b) $y = x^2 + C$

(c) $y = e^x + 2$

(d) $y = 3x + 4$

Answer: (b) $y = x^2 + C$

Solution: The general solution includes an arbitrary constant C, representing a family of solutions.

4. Solve: $\frac{dy}{dx} = 5y$, given $y(0) = 3$

   (a) $y = 3e^{5x}$

   (b) $y = 5e^{3x}$

   (c) $y = 3e^{-5x}$

   (d) $y = 5e^x$

   Answer: (a) $y = 3e^{5x}$

   Solution: General solution: $y = Ce^{5x}$. Use $y(0) = 3 \Rightarrow 3 = Ce^0 \Rightarrow C = 3$. Hence, $y = 3e^{5x}$

5. A differential equation has general solution $y = Ce^{-2x}$. If $y = 4$ when $x = 0$, the particular solution is:

   (a) $y = 4e^{2x}$

   (b) $y = 4e^{-2x}$

   (c) $y = 2e^{-2x}$

   (d) $y = 2e^{2x}$

   Answer: (b) $y = 4e^{-2x}$

   Solution: Substitute $x = 0 \Rightarrow y = C \cdot 1 = 4 \Rightarrow C = 4$. So the particular solution is $y = 4e^{-2x}$