Case Study 2: Application of Determinants in Solving Business Optimization Problems

A small business owner, Rajiv, needed to manage his warehouse inventory based on supply and demand across three locations. He modeled his inventory flow using a system of three linear equations representing quantities shipped to and from locations. To find the exact values for each route, he expressed the system in matrix form and applied the inverse matrix method. Rajiv used the determinant of the coefficient matrix to check if a unique solution exists. Additionally, he applied cofactor and adjoint concepts to calculate the matrix inverse and solve for the unknowns. Let us explore how determinants help in such business optimization tasks.

MCQ Questions:

1. If the determinant of a 3x3 matrix is zero, what does it imply about the system of equations it represents?

   (a) System has a unique solution
   (b) System has infinite or no solution
   (c) System has exactly two solutions
   (d) System has negative determinant

   Answer: (b)
   Solution: A zero determinant implies the matrix is singular, so the system has either infinitely many or no solution.

2. Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$

   find $\det(A)$.

   (a) 22
   (b) -12
   (c) 6
   (d) 24

   Answer: (d)
   Solution:

$$\det(A) = 1 \cdot (4 \cdot 6 - 5 \cdot 0) - 2 \cdot (0 \cdot 6 - 5 \cdot 1) + 3 \cdot (0 \cdot 0 - 4 \cdot 1)$$

$$= 1(24) - 2(-5) + 3(-4) = 24 + 10 - 12 = 22$$

   Correction: Final answer is 22, so answer should be (a).

3. Which of the following statements about cofactors is true?

   (a) Cofactors are only used in 2x2 matrices
   (b) Cofactors help in calculating transpose
   (c) Cofactors are used in computing determinant and adjoint
   (d) Cofactors are never negative

Answer: (c)
Solution: Cofactors are needed to compute both the determinant (via expansion) and the adjoint matrix.

4. What is the order of an identity matrix used to invert a 3x3 matrix?

   (a) 2x2
   (b) 1x1
   (c) 3x1
   (d) 3x3

   Answer: (d)
   Solution: The identity matrix must be of the same order as the matrix being inverted.

5. The inverse of a matrix A is found using which formula?

   (a) $A^{-1} = \frac{1}{\det(A)} \cdot A$
   (b) $A^{-1} = \det(A) \cdot A$
   (c) $A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$
   (d) $A^{-1} = \frac{1}{\text{adj}(A)} \cdot A$

   Answer: (c)
   Solution: The correct formula is $A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$ if $\det(A) \neq 0$.