Case Study 2

Ankita, a biology student, is analyzing the growth of bacteria in a petri dish. The population of bacteria at any time t (in hours) is modeled by the function $P(t) = e^{2t}$. She is also using logarithmic methods to convert this exponential model into a linear form for better analysis. During her study, she compares the rates of change in the population at different times and applies logarithmic differentiation to find precise values of population growth rate. Ankita also needs to understand whether these functions are differentiable and how their derivatives behave under different transformations. She creates composite functions and uses the chain rule and properties of logarithmic and exponential differentiation to analyze the data mathematically.

MCQ Questions

1. What is the derivative of $P(t) = e^{2t}$ with respect to t?

   (A) e^{2t}

   (B) $2e^{2t}$

   (C) $\ln(e^{2t})$

   (D) $\frac{1}{e^{2t}}$

   Answer: (B)
   Solution: Using chain rule, $\frac{d}{dt}e^{2t} = e^{2t} \cdot 2 = 2e^{2t}$.

2. If $f(t) = \ln(P(t))$ where $P(t) = e^{2t}$, then $f(t)$ is equal to:

   (A) $2t$

   (B) e^{2t}

   (C) $\ln 2t$

   (D) $\frac{1}{2t}$

   Answer: (A)
   Solution: Since $P(t) = e^{2t}$, then $\ln(P(t)) = \ln(e^{2t}) = 2t$ by logarithmic identity.

3. Find the second derivative of $f(t) = e^{2t}$ with respect to t.

   (A) $2e^{2t}$

   (B) $4e^{2t}$

   (C) e^{2t}

   (D) $8e^{2t}$

   Answer: (B)
   Solution: First derivative: $f'(t) = 2e^{2t}$, second derivative: $f''(t) = 2 \cdot 2e^{2t} = 4e^{2t}$.

4. Which of the following best describes the differentiability of the function $f(t) = e^{2t}$?

(A) Not differentiable at $t = 0$

(B) Differentiable at only positive values of t

(C) Differentiable at only negative values of t

(D) Differentiable for all real t

Answer: (D)
Solution: Exponential functions like e^{2t} are continuous and differentiable for all real values of t.

5. Using logarithmic differentiation, what is the derivative of $y = (e^{2t})^3$?

(A) $6e^{6t}$

(B) $3e^{2t}$

(C) e^{2t}

(D) $2e^{6t}$

Answer: (A)
Solution: $y = e^{6t}$, so $\frac{dy}{dt} = 6e^{6t}$.