

Case Study 2

A theme park engineering team is designing a new roller coaster that should provide a thrilling experience by carefully adjusting the heights and slopes of the ride. The height of the roller coaster at any point along the track is modeled by the function $h(x) = x^3 - 6x^2 + 9x + 5$, where x denotes the horizontal distance from the starting point (in meters) and $h(x)$ is the height (in meters). The engineers must determine where the ride is going uphill or downhill to avoid abrupt changes that could cause discomfort. They analyze the intervals where the function is increasing or decreasing using the concept of derivatives. This real-world application allows students to understand how mathematics ensures both safety and excitement in roller coaster design.

MCQ Questions:

1. Find the derivative $h'(x)$ of the function $h(x) = x^3 - 6x^2 + 9x + 5$.

(a) $3x^2 - 12x + 9$

(b) $x^2 - 3x + 1$

(c) $3x^2 + 6x + 9$

(d) $2x^2 - 6x + 1$

Answer: (A) $3x^2 - 12x + 9$

Solution: Differentiate: $h'(x) = 3x^2 - 12x + 9$

2. At what values of x is the function increasing?

(a) $x < 1$ and $x > 3$

(b) $x < 2$ only

(c) $x < 0$ only

(d) $x > 2$ only

Answer: (A) $x < 1$ and $x > 3$

Solution: Set $h'(x) = 0$:

$$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x = 1, 3$$

Use sign chart:

- $x < 1$: $h'(x) > 0$ (increasing)
- $1 < x < 3$: $h'(x) < 0$ (decreasing)
- $x > 3$: $h'(x) > 0$ (increasing)

3. What is the nature of the function on the interval $(1, 3)$?

(a) Increasing

(b) Decreasing

(c) Constant

(d) Not defined

Answer: (B) Decreasing

Solution: From sign chart of $h'(x)$, on $(1, 3)$ derivative is negative, so the function is decreasing.

4. Which of the following statements is true?

(a) The function has local maxima at $x = 1$

(b) The function has local minima at $x = 3$

(c) The function is always increasing

(d) The function has no turning points

Answer: (A) The function has local maxima at $x = 1$

Solution: Since $h'(x)$ changes sign from positive to negative at $x = 1$, local maxima exists there.

5. What is the value of $h(1)$?

(a) 9

(b) 12

(c) 15

(d) 11

Answer: (D) 11

Solution: $h(1) = (1)^3 - 6(1)^2 + 9(1) + 5 = 1 - 6 + 9 + 5 = 9$

Correction: This evaluates to 9, so correct answer is (A).