Case Study 5: Matrix Inversion in Business Forecasting

Raj is a financial analyst working on a business forecasting project. He uses matrices to represent various inputs like raw materials, labor, and production costs. To calculate outputs from these inputs, he represents the system as a matrix equation $AX = B$. Solving this equation requires finding the inverse of matrix A. He recalls that a square matrix has an inverse only if it is non-singular, i.e., its determinant is not zero. Raj double-checks his calculations and confirms the matrix is invertible. He uses the inverse to compute the result vector $X = A^{-1}B$, making predictions efficiently. This knowledge helps him advise businesses with accurate projections.

MCQ Questions:

1. Which condition must be satisfied for a matrix to have an inverse?

   (a) The matrix must be symmetric
   (b) The determinant must be zero
   (c) The matrix must be non-singular
   (d) The matrix must be skew-symmetric

   Answer: (c)
   Solution: A square matrix has an inverse if its determinant is non-zero, i.e., it is non-singular.

2. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, what is $\det(A)$?

   (a) 5
   (b) -2
   (c) 11
   (d) 7

   Answer: (d)
   Solution: $\det(A) = (2)(4) - (3)(1) = 8 - 1 = 7$

3. What is the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$?

   (a) $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$
   (b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} -2 & 1 \\ 1 & -0.5 \end{bmatrix}$
   (d) $\begin{bmatrix} 2 & -1 \\ -1.5 & 0.5 \end{bmatrix}$

   Answer: (a)
   Solution:

$$\text{Adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}, \quad \det(A) = -2$$

$$A^{-1} = \frac{1}{-2} \cdot \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

4. Which of the following matrix operations is not always defined?

   (a) Matrix addition of two matrices of same order

   (b) Multiplication of a matrix by a scalar

   (c) Multiplication of two matrices of compatible order

   (d) Finding inverse of a singular matrix

   Answer: (d)
   Solution: A singular matrix (determinant zero) does not have an inverse.

5. If $A \cdot A^{-1} = I$, what is I?

   (a) A null matrix

   (b) A unit matrix

   (c) A symmetric matrix

   (d) A skew-symmetric matrix

   Answer: (b)
   Solution: I is the identity matrix where all diagonal elements are 1 and others are 0.