Case Study 1:

In a senior secondary school, the principal decided to analyze the interaction pattern between teachers and students for better management of academic resources. The set of all teachers in the school is denoted by $T = \{t_1, t_2, t_3, t_4\}$, and the set of all students in a particular class is denoted by $S = \{s_1, s_2, s_3, s_4, s_5\}$. A relation R from T to S is defined as (t_i, s_j) if teacher t_i interacts with student s_j during the week. The following pairs are included in R: $(t_1, s_1), (t_1, s_2), (t_2, s_2), (t_2, s_3), (t_3, s_4), (t_4, s_5)$. Using this context, answer the following multiple-choice questions:

1. What is the domain of the relation R?

   (a) $\{t_1, t_2, t_3, t_4\}$

   (b) $\{s_1, s_2, s_3, s_4, s_5\}$

   (c) $\{t_1, t_2, t_3\}$

   (d) $\{s_1, s_2, s_3\}$

Answer: (a) $\{t_1, t_2, t_3, t_4\}$

Solution: The domain of a relation consists of all first elements of the ordered pairs. Here, all teachers appear in at least one pair, so domain is $\{t_1, t_2, t_3, t_4\}$.

2. What is the range of the relation R?

   (a) $\{t_1, t_2, t_3, t_4\}$

   (b) $\{s_1, s_2, s_3, s_4, s_5\}$

   (c) $\{s_1, s_2, s_3, s_4, s_5\}$

   (d) $\{s_1, s_2, s_3, s_4, s_5, t_1, t_2, t_3, t_4\}$

Answer: (c) $\{s_1, s_2, s_3, s_4, s_5\}$

Solution: The range is the set of all second elements of the ordered pairs, which are all the students: $\{s_1, s_2, s_3, s_4, s_5\}$.

3. Which of the following best describes the codomain of the relation R?

(a) $\{t_1, t_2, t_3, t_4\}$

(b) $\{s_1, s_2, s_3, s_4, s_5\}$

(c) $\{s_1, s_2, s_3\}$

(d) $\{t_1, t_2, t_3\}$

Answer: (b) $\{s_1, s_2, s_3, s_4, s_5\}$

Solution: The codomain is the set to which the second elements of ordered pairs belong. Here, it is the set of all students, $\{s_1, s_2, s_3, s_4, s_5\}$.

4. How many elements are in the relation R?

   (a) 4
   (b) 5
   (c) 6
   (d) 7

Answer: (c) 6

Solution: Counting the given ordered pairs: $(t_1, s_1), (t_1, s_2), (t_2, s_2), (t_2, s_3), (t_3, s_4),$
we have 6 pairs.

5. Is the relation R a function from T to S?

   (a) Yes

   (b) No

   (c) Cannot be determined

   (d) Only if T has more elements

Answer: (b) No

Solution: A relation is a function if each element in the domain is paired with exactly one element in the codomain. Here, t_1 is paired with both s_1 and s_2, and t_2 is paired with both s_2 and s_3, so R is not a function.