Case Study 1: Understanding Matrix Operations

Ritika is preparing for her Grade 12 board examination and is revising the chapter on Matrices. She comes across a practical problem related to image transformations in computer graphics. The transformations are represented using matrices, and she realizes how important it is to understand matrix multiplication, identity matrices, and the concept of inverse matrices. During her revision, she also encounters symmetric and skew-symmetric matrices, and wonders how to identify and differentiate them. To help herself revise, she decides to solve some conceptual questions to test her understanding of matrix operations and properties.

MCQ Questions:

1. Let $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. What is AB?

   (a) $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$

   Answer: (a)

   Solution: $AB = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2(1)+0(0) & 2(2)+0(1) \\ 1(1)+3(0) & 1(2)+3(1) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$

2. Which of the following matrices is symmetric?

   (a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$

   Answer: (b)

   Solution: A matrix is symmetric if $A = A^T$. Option (b) satisfies this condition as $A = A^T$.

3. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. What is A^T?

   (a) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Answer: (a)
Solution: The transpose of a matrix is obtained by interchanging its rows and columns: $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

4. Which of the following statements is true about matrix multiplication?

   (a) Matrix multiplication is always commutative.

   (b) Matrix multiplication is associative but not commutative.

   (c) Matrix multiplication is distributive over addition and commutative.

   (d) Matrix multiplication is neither associative nor distributive.

   Answer: (b)
   Solution: Matrix multiplication is associative and distributive over addition but not commutative in general. That is, $AB \neq BA$ for most matrices.

5. Which matrix is the identity matrix of order 2?

   (a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

   Answer: (c)
   Solution: The identity matrix has 1s on the diagonal and 0s elsewhere. Thus, for order 2, it is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$