Case Study Based Questions
Chapter: Inverse Trigonometric Functions

Case Study 1:
Ravi, a Class 12 student, is preparing for his final mathematics examination. While studying inverse trigonometric functions, he encounters a variety of problems involving the domain, range, and principal values of these functions. During a practice session, his teacher gives him a set of questions to test his understanding of the inverse trigonometric identities and their graphical interpretations. Ravi realizes that understanding the principal value branches and their respective domains is critical in solving such problems correctly. He also learns that many expressions involving inverse trigonometric functions can be simplified using identities such as $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and others. The following questions are based on Ravi's learning journey and the concepts discussed.

MCQ Questions:

1. The value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is:

   (a) $\frac{\pi}{2}$
   (b) π
   (c) 0
   (d) $\frac{\pi}{3}$

   Answer: (a)
   Solution: We know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all $x \in [-1, 1]$. Therefore, $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$.

2. The principal value of $\tan^{-1}(\sqrt{3})$ is:

   (a) $\frac{\pi}{6}$
   (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{4}$
   (d) $\frac{\pi}{2}$

   Answer: (b)
   Solution: $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ since $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ and $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1} x$.

3. The domain of $\sec^{-1} x$ is:

   (a) $(-\infty, -1] \cup [1, \infty)$

(b) $[-1, 1]$

(c) $(0, \pi)$

(d) $(0, \infty)$

Answer: (a)
Solution: The function $\sec^{-1} x$ is defined for all real numbers x such that $|x| \geq 1$. Hence, the domain is $(-\infty, -1] \cup [1, \infty)$.

4. The graph of $\sin^{-1} x$ lies within:

(a) $x \in [-\frac{\pi}{2}, \frac{\pi}{2}], y \in [-1, 1]$

(b) $x \in [-1, 1], y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

(c) $x \in [-1, 1], y \in [0, \pi]$

(d) $x \in [-\pi, \pi], y \in [-1, 1]$

Answer: (b)
Solution: For $\sin^{-1} x$, the domain is $[-1, 1]$ and the range (principal value branch) is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. So, the graph lies in $x \in [-1, 1], y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

5. The identity $\tan^{-1} x + \cot^{-1} x$ equals:

(a) π

(b) $\frac{\pi}{2}$

(c) 0

(d) x

Answer: (b)
Solution: For all $x > 0$, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$. This is a standard identity derived from the relationship between the two inverse functions.