Case Study 5: Designing a Garden – Maxima and Minima in Area Optimization

A landscape designer is tasked with planning a rectangular garden using 120 meters of fencing. One side of the garden will be built along an existing wall, so fencing is needed only for the other three sides. The designer aims to maximize the area of the garden to provide the most space for plants. To achieve this, she applies the concept of maxima and minima using derivatives. By forming a function for the area in terms of one variable and finding its derivative, she can identify the dimensions that yield the maximum possible area. This real-world problem directly applies concepts from calculus, especially optimization using the first and second derivative tests.

MCQ Questions:

1. If x is the length perpendicular to the wall, what is the expression for the area $A(x)$?

   (a) $A(x) = x(120 - x)$

   (b) $A(x) = x(60 - x)$

   (c) $A(x) = x(120 - 2x)$

   (d) $A(x) = x(60 - 2x)$

   Answer: (C) $A(x) = x(120 - 2x)$
   Solution: Let x be length perpendicular to wall. Then remaining fencing $= 120 - 2x$, so length along wall $= 120 - 2x$, hence area $A(x) = x(120 - 2x)$

2. What is the first derivative $A'(x)$ of $A(x) = x(120 - 2x)$?

   (a) $A'(x) = 120 - 4x$

   (b) $A'(x) = 120 - 2x$

   (c) $A'(x) = 60 - 2x$

   (d) $A'(x) = 120x - 2$

   Answer: (A) $A'(x) = 120 - 4x$
   Solution: $A(x) = 120x - 2x^2$, so $A'(x) = 120 - 4x$

3. For what value of x is the area maximum?

   (a) $x = 20$

   (b) $x = 25$

   (c) $x = 30$

   (d) $x = 15$

   Answer: (C) $x = 30$
   Solution: Set $A'(x) = 0 \Rightarrow 120 - 4x = 0 \Rightarrow x = 30$

4. What is the maximum area of the garden?

   (a) 900 m^2

   (b) 1000 m^2

   (c) 1800 m^2

   (d) 1200 m^2

   Answer: (C) 1800 m^2
   Solution: At $x = 30$, length along wall $= 120 - 2x = 60$, so $A = 30 \times 60 = 1800$ m^2

5. Which test confirms that this point gives a maximum area?

   (a) First Derivative Test

   (b) Second Derivative Test

   (c) Both tests

   (d) None of these

   Answer: (C) Both tests
   Solution: $A'(x)$ changes sign around $x = 30$ and $A''(x) = -4 < 0$, confirming maxima by both tests