Limits and Continuity – Set 1

Instructions for Solving the DPP (Daily Practice Problems)

1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
  - **Level–1**: JEE Main oriented
  - **Level–2**: Mixed Main + Advanced
  - **Level–3**: JEE Advanced oriented

2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
   - Key concept involved
   - Formula used
   - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**
- Follow the recommended per-question time:
  - Single Correct / Objective: **1–2 minutes**
  - Numerical Value: **2–3 minutes**
  - Integer Type: **3–4 minutes**
  - Advanced Multi-Correct: **4–6 minutes**
  - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (*) and revisit after finishing the DPP.

4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.
- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with the official solution or teacher's method.
- Build endurance by solving at least one DPP in exam-like conditions daily.

5. Evaluation Guidelines

- Award yourself:
  - **+4 / -1** for JEE Main pattern questions.
  - **Partial marking** for JEE Advanced style multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
  - Chapters with highest accuracy
  - Chapters needing revision
  - Time taken per DPP
  - Common error types

6. Weekly Review Checklist

- Reattempt the unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

By: www.udgamwelfarefoundation.com (helping students since 2012)

Daily Practice Problems

Multiple Choice Questions

1. Evaluate the following limit:

$$\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2 \sin x}$$

   (a) $\frac{1}{3}$

   (b) $-\frac{1}{3}$

   (c) $\frac{1}{6}$

   (d) $-\frac{1}{6}$

2. Prove that the following limit equals 2:

$$\lim_{x\to\infty} \frac{(3x-1)(2x+5)}{(x-3)(3x+7)}$$

   The value of the limit is:

   (a) 1

   (b) 2

   (c) 3

   (d) 6

3. Show that the following inequality is true:

$$\lim_{x\to -2} \frac{\tan \pi x}{x+2} + \lim_{x\to\infty} \left(1+\frac{1}{x^2}\right)^x > 3$$

   The value of the left hand side is:

   (a) $\pi + 1$

   (b) π

   (c) 2π

   (d) $\pi + 2$

4. If $f(2) = 4$ and $f'(2) = 1$, then $\lim_{x\to 2} \frac{xf(2) - 2f(x)}{x-2}$ equals:

   (a) 2

   (b) 4

   (c) 6

   (d) 0

5. Find the value of the two-sided limit (if it exists):

$$\lim_{x\to 2} \frac{|x^2-4|}{x-2}$$

   (a) 4

   (b) 0

   (c) 2

   (d) Does not exist

6. Find the value of $\lim_{n\to\infty} \left[\frac{1}{n^3+1} + \frac{4}{n^3+1} + \frac{9}{n^3+1} + ... + \frac{n^2}{n^3-1}\right]$

   (a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) 0

(d) 1

7. Evaluate the limit:

$$\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\cdots\cos\left(\frac{x}{2^n}\right)$$

(a) $\frac{\cos x}{x}$

(b) $\frac{\sin x}{x}$

(c) $\sin x$

(d) 1

8. Evaluate the limit:

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{2}{x}} \quad ;(a,b,c>0)$$

(a) $a^2b^2c^2$

(b) $(abc)^3$

(c) $(abc)^{\frac{2}{3}}$

(d) $(a+b+c)/3$

9. Evaluate the limit:

$$\lim_{x\to\infty} \frac{x^2\sin\frac{1}{x}-x}{1-|x|}$$

(a) 0

(b) 1

(c) -1

(d) $\frac{1}{2}$

10. Evaluate the limit:

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2+100}$$

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 0

11. Evaluate the limit:

$$\lim_{n\to\infty} \frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}+\cdots+\frac{n}{1-n^2}$$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 1

(d) -1

12. Evaluate the limit:

$$\lim_{x\to-\infty} \frac{x^4\sin(\frac{1}{x})+x^2}{1+|x|^3}$$

(a) 1

(b) -1

(c) 0

(d) -2

13. Evaluate the limit:

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) 1

(d) 0

Integer Type Questions

14. Evaluate the value of a, b and c such that:

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$$

Find the value of $a+b+c$.

15. Evaluate the limit:

$$\lim_{x\to\frac{\pi}{3}} \frac{\tan^3 x - 3\tan x}{\cos(x+\frac{\pi}{6})}$$

16. Find the value of k if the following holds:

$$\lim_{x\to 0} \frac{\log(a+x)-\log a}{x} + k\lim_{x\to e} \frac{\log x - 1}{x-e} = 1$$

Find the value of k/e in terms of a.

17. Let $f : R \to R$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$. Find the value of:

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x-2}$$

18. Evaluate the limit:

$$\lim_{x\to 3} \frac{\sqrt{1-\cos(x^2-10x+21}}{(x-3)}$$

Find the absolute value of the limit.