

# Limits and Trigonometry – Set 3

## Instructions for Solving the DPP (Daily Practice Problems)

### 1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
  - **Level-1:** JEE Main oriented
  - **Level-2:** Mixed Main + Advanced
  - **Level-3:** JEE Advanced oriented

### 2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
  - Key concept involved
  - Formula used
  - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

### 3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**
- Follow the recommended per-question time:
  - Single Correct / Objective: **1–2 minutes**
  - Numerical Value: **2–3 minutes**
  - Integer Type: **3–4 minutes**
  - Advanced Multi-Correct: **4–6 minutes**
  - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (\*) and revisit after finishing the DPP.

### 4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.
- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with the official solution or teacher's method.
- Build endurance by solving at least one DPP in exam-like conditions daily.

## 5. Evaluation Guidelines

- Award yourself:
  - **+4 / -1** for JEE Main pattern questions.
  - **Partial marking** for JEE Advanced style multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
  - Chapters with highest accuracy
  - Chapters needing revision
  - Time taken per DPP
  - Common error types

## 6. Weekly Review Checklist

- Reattempt the unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

By: [www.udgamwelfarefoundation.com](http://www.udgamwelfarefoundation.com) (helping students since 2012)

# Daily Practice Problems

## Multiple Choice Questions

1. Show that the following limit equals  $e$ :

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$$

The value of the limit is:

- (a) 1
  - (b)  $e$
  - (c)  $\frac{1}{e}$
  - (d)  $e^2$
2. Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{x^9 - 1}{x^{14} - 1}$$

- (a)  $\frac{9}{14}$
  - (b)  $\frac{14}{9}$
  - (c) 1
  - (d) 0
3. Evaluate the following limit:

$$\lim_{x \rightarrow \tan^{-1}(3)} \frac{\tan^2 x - 2 \tan x - 3}{\tan^2 x - 4 \tan x + 3}$$

- (a) 1
  - (b) 3
  - (c) 2
  - (d) 4
4. Evaluate the limit:

$$\lim_{x \rightarrow 2} \frac{3^{\frac{x}{2}} - 3}{3^x - 9}$$

- (a)  $\frac{1}{4}$
  - (b)  $\frac{1}{2}$
  - (c) 1
  - (d)  $\frac{1}{3}$
5. Evaluate the limit:

$$\lim_{\alpha \rightarrow 0} \frac{\sin^2 \alpha - \sin^2 \beta}{\alpha^2 - \beta^2}$$

- (a)  $\sin \beta \cos \beta$
  - (b) 1
  - (c)  $\sin 2\beta$
  - (d)  $\frac{\sin 2\beta}{2\beta}$
6. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$$

- (a) 2
- (b)  $\sqrt{2}$
- (c) 1

(d)  $-2$

7. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{\sin \frac{2}{3^x}}{\sin \frac{3}{2^x}}$$

(a)  $1$

(b)  $\frac{2}{3}$

(c)  $\frac{3}{2}$

(d)  $0$

8. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$$

(a)  $\frac{1}{24}$

(b)  $\frac{1}{120}$

(c)  $\frac{1}{6}$

(d)  $0$

9. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{(1+x)^5 - 1}{3x + 5x^2}$$

(a)  $\frac{5}{3}$

(b)  $1$

(c)  $\frac{3}{5}$

(d)  $5$

10. Evaluate the limit:

$$\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\sin \theta - \cos \theta}{\theta - \frac{\pi}{4}}$$

(a)  $\sqrt{2}$

(b)  $\frac{1}{\sqrt{2}}$

(c)  $1$

(d)  $2$

11. Evaluate the limit (Let  $a \neq 0$ ):

$$\lim_{x \rightarrow a} \left( \frac{\sin x}{\sin a} \right)^{\frac{1}{(x-a)}}$$

(a)  $e^{\cot a}$

(b)  $e^{\tan a}$

(c)  $\cot a$

(d)  $1$

12. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{x}{\tan^{-1} 2x}$$

(a)  $1$

(b)  $\frac{1}{2}$

(c)  $2$

(d)  $0$

13. Evaluate the limit:

$$\lim_{n \rightarrow \infty} (4^n + 5^n)^{\frac{1}{n}}$$

- (a) 4
- (b) 5
- (c) 9
- (d) 1

14. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \sqrt{\frac{x + \sin x}{x - \cos x}}$$

- (a) 1
- (b) 0
- (c)  $\sqrt{2}$
- (d) Does not exist

15. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \frac{n^p \sin^2(n!)}{n^{1-p}(1 + \frac{1}{n})}$$

for  $p < 0$ .

- (a) 1
- (b) 0
- (c)  $\infty$
- (d)  $\frac{1}{2}$

16. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

- (a)  $\frac{1}{2\sqrt{2}}$
- (b)  $\frac{1}{4\sqrt{2}}$
- (c)  $\frac{1}{8\sqrt{2}}$
- (d)  $\frac{1}{\sqrt{2}}$

## Integer Type Questions

17. Evaluate the limit:

$$\lim_{n \rightarrow \infty} n \cos\left(\frac{\pi}{4n}\right) \sin\left(\frac{\pi}{4n}\right)$$

Find the value of the limit.

18. Evaluate the limit:

$$\lim_{x \rightarrow 3} \frac{(x^3 + 27) \ln(x - 2)}{x^2 - 9}$$

19. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

Find the value of the limit as a fraction in simplest form  $p/q$ , and give the value of  $p + q$ .

20. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin[\cos x]}{1 + [\cos x]}, \text{ where } [.] \text{ denotes the greatest integer function.}$$

21. Evaluate the limit:

$$\lim_{x \rightarrow 0} \left[ \frac{\sin x}{x} \right] \text{ where } [.] \text{ denotes the greatest integer function.}$$