

Limits and Series – Set 4

Instructions for Solving the DPP (Daily Practice Problems)

1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
 - **Level-1:** JEE Main oriented
 - **Level-2:** Mixed Main + Advanced
 - **Level-3:** JEE Advanced oriented

2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
 - Key concept involved
 - Formula used
 - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**
- Follow the recommended per-question time:
 - Single Correct / Objective: **1–2 minutes**
 - Numerical Value: **2–3 minutes**
 - Integer Type: **3–4 minutes**
 - Advanced Multi-Correct: **4–6 minutes**
 - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (*) and revisit after finishing the DPP.

4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.
- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with the official solution or teacher's method.
- Build endurance by solving at least one DPP in exam-like conditions daily.

5. Evaluation Guidelines

- Award yourself:
 - **+4 / -1** for JEE Main pattern questions.
 - **Partial marking** for JEE Advanced style multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
 - Chapters with highest accuracy
 - Chapters needing revision
 - Time taken per DPP
 - Common error types

6. Weekly Review Checklist

- Reattempt the unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

By: www.udgamwelfarefoundation.com (helping students since 2012)

Daily Practice Problems

Multiple Choice Questions

1. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2}$$

- (a) $\frac{1}{18}$
- (b) $\frac{1}{36}$
- (c) $\frac{\sqrt{3}}{18}$
- (d) $\frac{1}{72}$

2. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x+1}$$

- (a) e^{-2}
- (b) e
- (c) e^{-3}
- (d) e^2

3. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{(\cos x)^{\frac{1}{2}} - (\cos x)^{\frac{1}{3}}}{\sin^2 x}$$

- (a) $\frac{-1}{12}$
- (b) $\frac{-1}{14}$
- (c) $\frac{1}{6}$
- (d) $\frac{-1}{3}$

4. Evaluate the limit:

$$\lim_{x \rightarrow 1} \frac{1 + \log x - x}{1 - 2x + x^2}$$

- (a) $\frac{1}{2}$
- (b) $\frac{-1}{2}$
- (c) $\frac{-1}{5}$
- (d) $\frac{-1}{3}$

5. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$$

- (a) $\frac{1}{6}$
- (b) $\frac{2}{3}$
- (c) $\frac{1}{3}$
- (d) $\frac{5}{9}$

6. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \left[\frac{1}{1-n^4} + \frac{8}{1-n^4} + \cdots + \frac{n^3}{1-n^4} \right]$$

- (a) $\frac{-1}{2}$
- (b) $\frac{-1}{3}$
- (c) $\frac{-1}{4}$
- (d) $\frac{1}{4}$

7. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{2 \sin(x - \frac{\pi}{3})}{1 - 2 \cos x}$$

- (a) $\frac{2}{\sqrt{5}}$
- (b) $\frac{3}{\sqrt{3}}$
- (c) $\frac{2}{\sqrt{3}}$
- (d) $\frac{2}{\sqrt{7}}$

8. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$$

- (a) $\frac{3}{4}$
- (b) -1
- (c) $\frac{3}{5}$
- (d) $\frac{-3}{4}$

9. Evaluate the limit:

$$\lim_{x \rightarrow 1} \left[\left(\frac{4}{x^2 - x^{-1}} - \frac{1 - 3x + x^2}{1 - x^3} \right)^{-1} + 3 \frac{x^4 - 1}{x^3 - x^{-1}} \right]$$

- (a) 3
- (b) -3
- (c) 5
- (d) 4

10. The value of the limit is:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \text{up to } n \text{ terms} \right)$$

- (a) $\frac{-1}{2}$
- (b) 0
- (c) $\frac{1}{2}$
- (d) $\frac{1}{3}$

11. If $\lim_{x \rightarrow 0} (1 + ax)^{\frac{b}{x}} = e^4$, where a and b are natural numbers, then ab equals:

- (a) 4
- (b) 6

- (c) 8
- (d) 14

12. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{\sqrt{a+x} - \sqrt{a}}$$

- (a) $2\sqrt{a^2} \log a$
- (b) $2\sqrt{3a} \log a$
- (c) $2\sqrt{a} \log a$
- (d) $\sqrt{a} \log a$

13. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$. The value of the limit is:

$$\lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$$

- (a) $\frac{53}{31}$
- (b) $\frac{53}{3}$
- (c) $\frac{53}{13}$
- (d) $\frac{5}{3}$

14. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$$

- (a) $\frac{3}{2}$
- (b) $\frac{2}{5}$
- (c) $\frac{3}{4}$
- (d) $\frac{-3}{2}$

15. If f is a strictly increasing differentiable function with $f(0) = 0$, then the value of the limit is:

$$\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$$

- (a) -2
- (b) -3
- (c) 1
- (d) -1

Integer Type Questions

16. Find the integral value of n for which the following limit is a finite non-zero constant:

$$\lim_{x \rightarrow 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$$

17. If $[x]$ denotes the greatest integer $\leq x$ then evaluate:

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} \{[1^2x] + [2^2x] + [3^2x] + \cdots + [n^2x]\}$$