

Limits and Series – Set 2

Instructions for Solving the DPP (Daily Practice Problems)

1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
 - **Level-1:** JEE Main oriented
 - **Level-2:** Mixed Main + Advanced
 - **Level-3:** JEE Advanced oriented

2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
 - Key concept involved
 - Formula used
 - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**
- Follow the recommended per-question time:
 - Single Correct / Objective: **1–2 minutes**
 - Numerical Value: **2–3 minutes**
 - Integer Type: **3–4 minutes**
 - Advanced Multi-Correct: **4–6 minutes**
 - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (*) and revisit after finishing the DPP.

4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.
- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with the official solution or teacher's method.
- Build endurance by solving at least one DPP in exam-like conditions daily.

5. Evaluation Guidelines

- Award yourself:
 - **+4 / -1** for JEE Main pattern questions.
 - **Partial marking** for JEE Advanced style multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
 - Chapters with highest accuracy
 - Chapters needing revision
 - Time taken per DPP
 - Common error types

6. Weekly Review Checklist

- Reattempt the unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

By: www.udgamwelfarefoundation.com (helping students since 2012)

Daily Practice Problems

Multiple Choice Questions

1. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n} \right)$$

- (a) 1
- (b) $\frac{1}{3}$
- (c) $\frac{1}{2}$
- (d) 0

2. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \prod_{r=2}^n \left(\frac{r^3 + 1}{r^3 - 1} \right)$$

- (a) $\frac{3}{2}$
- (b) 1
- (c) $\frac{2}{3}$
- (d) 2

3. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \prod_{r=3}^n \frac{r^3 + 2^3}{r^3 - 2^3}$$

- (a) $\frac{7}{4}$
- (b) $\frac{5}{2}$
- (c) $\frac{7}{2}$
- (d) $\frac{7}{3}$

4. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{r^4 + r^2 + 1}$$

- (a) $\frac{1}{2}$
- (b) $\frac{1}{4}$
- (c) 1
- (d) $\frac{3}{4}$

5. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \cot^{-1} \left(r^2 + \frac{3}{4} \right)$$

- (a) $\frac{\pi}{2}$
- (b) $\tan^{-1}(2)$
- (c) $\frac{\pi}{4}$
- (d) $\tan^{-1}\left(\frac{1}{2}\right)$

6. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \cot^{-1}(2r^2)$$

- (a) 0
- (b) $\frac{\pi}{2}$
- (c) $\frac{\pi}{4}$
- (d) $\tan^{-1}(2)$

7. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \prod_{r=1}^n \cos\left(\frac{x}{2^r}\right)$$

- (a) $\frac{\sin 2x}{2x}$
- (b) $\frac{x}{\sin x}$
- (c) $\frac{\sin x}{x}$
- (d) $\cos x$

8. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$$

- (a) 1
- (b) 0
- (c) $\frac{1}{2}$
- (d) Does not exist

9. Evaluate the limit:

$$\lim_{x \rightarrow \infty} x \left[\tan^{-1} \left(\frac{x+1}{x+2} \right) - \frac{\pi}{4} \right]$$

- (a) $\frac{1}{2}$
- (b) -1
- (c) 0
- (d) $-\frac{1}{2}$

10. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x \tan x}$$

- (a) $\ln 2 + \ln 3$
- (b) $\ln(6)$
- (c) $\ln 3 \cdot \ln 2$
- (d) 1

11. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$$

- (a) $\ln 2$
- (b) $\ln 4$
- (c) 2
- (d) 0

12. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{(1 - \tan \frac{x}{2})(1 - \sin x)}{(1 + \tan \frac{x}{2})(\pi - 2x)^3}$$

- (a) $\frac{1}{16}$
- (b) $\frac{1}{32}$
- (c) $\frac{1}{8}$
- (d) 0

13. The value of the limit is:

$$\lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$$

- (a) $\frac{1}{3}$
- (b) $\frac{2}{3}$
- (c) 1
- (d) 0

14. Evaluate the limit:

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$$

- (a) 0
- (b) $\frac{1}{\sqrt{\pi}}$
- (c) $\frac{1}{\sqrt{2\pi}}$
- (d) $\frac{1}{2}$

15. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan x \log_e(\sin x)$$

- (a) 1
- (b) 0
- (c) -1
- (d) $\frac{1}{2}$

16. Find the limit:

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

- (a) 1
- (b) $\frac{1}{2}$
- (c) 0
- (d) ∞

Integer Type Questions

17. Evaluate the limit (if it exists):

$$\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$$

18. Evaluate the limit and find the coefficient of x :

$$\lim_{n \rightarrow \infty} \frac{[x] + [3x] + [5x] + \dots + [(2n-1)x]}{n^2}$$

19. Evaluate the limit:

$$\lim_{x \rightarrow \infty} ((x-1)(x-2)(x+5))^{\frac{1}{3}} - x$$

Find the value of the limit as a fraction in simplest form p/q , and give the value of $p+q$.