Instructions for Solving the DPP (Daily Practice Problems)

1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
  - **Level–1**: JEE Main oriented
  - **Level–2**: Mixed Main + Advanced
  - **Level–3**: JEE Advanced oriented

2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
   - Key concept involved
   - Formula used
   - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**.
- Recommended time per question:
  - Single Correct / Objective: **1–2 minutes**
  - Numerical Value: **2–3 minutes**
  - Integer Type: **3–4 minutes**
  - Advanced Multi-Correct: **4–6 minutes**
  - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (*) and revisit them later.

4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.

- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with official or teacher solutions.
- Build endurance by solving at least one DPP in exam-like conditions daily.

5. Evaluation Guidelines

- Award yourself:
  - **+4 / -1** for JEE Main pattern questions.
  - **Partial marking** for JEE Advanced multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
  - Chapters with highest accuracy
  - Chapters needing revision
  - Time taken per DPP
  - Common error types

6. Weekly Review Checklist

- Reattempt unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

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Set 3 - Limits and Continuity

Subjective Questions

1. Determine the constants a, b, c for

$$f(x) = \begin{cases} (1+ax)^{\frac{1}{x}}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{(x+c)^{\frac{1}{3}} - 1}{(x+1)^{\frac{1}{2}} - 1}, & \text{if } x > 0 \end{cases}$$

   to be continuous.

2. Find $\lim_{x\to 2} \frac{|x^2-4|}{x-2}$

3. Evaluate $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

4. If

$$f(x) = \begin{cases} \sin x, & x \neq n\pi,\ n = 0, \pm 1, \pm 2 \ldots \\ 2, & \text{otherwise} \end{cases}$$

   and

$$g(x) = \begin{cases} x^2 + 1, & \text{if } x \neq 0, 2 \\ 4, & \text{if } x = 0 \\ 5, & \text{if } x = 2 \end{cases}$$

   find $\lim_{x\to 0} g[f(x)]$.

5. If $f(9) = 9,\ f'(9) = 4$ then find $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$.

Multiple Choice Questions

6. If $G(x) = -\sqrt{25 - x^2}$, then $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}$ has the value:

   (a) $\frac{1}{\sqrt{24}}$
   (b) $\frac{1}{5}$
   (c) $-\sqrt{24}$
   (d) none of these

7. If $f(a) = 2,\ f'(a) = 1,\ g(a) = -1,\ g'(a) = 2$, then value of $\lim_{x\to a} \frac{g(a)f(a)-g(a)f(x)}{x-a}$ is:

   (a) -5
   (b) $\frac{1}{5}$
   (c) 5
   (d) none of these

8. $\lim_{x\to\infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + \cdots + \frac{n}{1-n^2} \right\}$ is equal to:

   (a) 0

(b) $\frac{-1}{2}$
(c) $\frac{1}{2}$
(d) none of these

9. If
$$f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{if } x \neq 0 \\ 0, & \text{if } [x] = 0 \end{cases}$$
where $[x]$ is the greatest integer $\leq x$, then $\lim_{x\to 0} f(x)$ equals:

(a) 1
(b) 0
(c) -1
(d) none of these

10. The value of $\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos^2 x)}}{x}$:

(a) 1
(b) -1
(c) 0
(d) none of these

11. $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals:

(a) $1+\sqrt{5}$
(b) $-1+\sqrt{5}$
(c) $-1+\sqrt{2}$
(d) $1+\sqrt{2}$

12. $\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x}$:

(a) exists and equals $\sqrt{2}$
(b) exists and equals $-\sqrt{2}$
(c) does not exist because $x-1 \to 0$
(d) does not exist because LHL $\neq$ RHL

13. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is:

(a) 2
(b) -2
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$

14. For $x \in \mathbb{R}$, $\lim_{x\to\infty} \left(\frac{x-3}{x+3}\right)^x$ equals:

(a) e
(b) e^{-1}

(c) e^{-5}

(d) e^{5}

15. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals:

(a) $-\pi$

(b) π

(c) $\frac{\pi}{2}$

(d) 1