

Limits and Calculus Applications – Set 5

Instructions for Solving the DPP (Daily Practice Problems)

1. Purpose of the DPP

- This DPP is designed to strengthen concept clarity for both **JEE Main** and **JEE Advanced**.
- Problems are arranged in increasing order of difficulty:
 - **Level-1:** JEE Main oriented
 - **Level-2:** Mixed Main + Advanced
 - **Level-3:** JEE Advanced oriented

2. How to Attempt the DPP

1. Read the theory from your notes before attempting the problems.
2. Do not jump between questions; solve sequentially unless instructed otherwise.
3. For each question, write:
 - Key concept involved
 - Formula used
 - Corrected approach if you made an error
4. Maintain a separate **DPP Mistake Notebook**.

3. Recommended Time Allocation

- Total time per DPP: **45–60 minutes**
- Follow the recommended per-question time:
 - Single Correct / Objective: **1–2 minutes**
 - Numerical Value: **2–3 minutes**
 - Integer Type: **3–4 minutes**
 - Advanced Multi-Correct: **4–6 minutes**
 - Paragraph (Advanced): **6–8 minutes**
- Mark questions exceeding time limit with a star (*) and revisit after finishing the DPP.

4. Best Practices for Scoring Higher

- Focus on accuracy first, then speed.
- Review every calculation step—most mistakes arise from small algebraic slips.
- Solve advanced problems only after finishing Main-level questions for the chapter.
- Revise solved DPPs weekly and note repeating mistake patterns.
- Use short notes for formulas, special results, and commonly used approximations.
- After solving, compare your approach with the official solution or teacher's method.
- Build endurance by solving at least one DPP in exam-like conditions daily.

5. Evaluation Guidelines

- Award yourself:
 - **+4 / -1** for JEE Main pattern questions.
 - **Partial marking** for JEE Advanced style multi-correct.
- Maintain a cumulative score record for every DPP set.
- Track:
 - Chapters with highest accuracy
 - Chapters needing revision
 - Time taken per DPP
 - Common error types

6. Weekly Review Checklist

- Reattempt the unsolved or incorrect problems from the past 5–7 DPPs.
- Update your formula sheet and mistake notebook.
- Solve at least one mixed-topic DPP to test retention.

By: www.udgamwelfarefoundation.com (helping students since 2012)

Daily Practice Problems

Multiple Choice Questions

1. Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sin \left[\pi \sqrt{n^2 + 1} \right]$$

- (a) ∞
- (b) 0
- (c) Does not exist
- (d) -1

2. The value of the limit is:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} \right)$$

- (a) $\frac{1}{2}$
- (b) $\frac{1}{3}$
- (c) $\frac{2}{3}$
- (d) $\frac{1}{4}$

3. The value of the limit is:

$$\lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}}$$

- (a) $e^{-\frac{1}{3}}$
- (b) $e^{-\frac{2}{3}}$
- (c) e^{-1}
- (d) e^{-2}

4. Given $f'(2) = 6$ and $f'(1) = 4$, the value of the limit is:

$$\lim_{h \rightarrow 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$$

- (a) Does not exist
- (b) $\frac{-3}{2}$
- (c) 3
- (d) 2

5. If $f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)}$, where $0 < a < \frac{1}{2}$, is:

- (a) $\frac{-3}{2x}$
- (b) $\frac{-3(1+x)}{2}$
- (c) $\frac{-3}{2}$
- (d) $\frac{-3(1-a^2)}{2}$

6. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ is:

- (a) 3

- (b) -1
- (c) 0
- (d) 1

7. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \left[x \tan x - \left(\frac{\pi}{2} \right) \sec x \right]$$

- (a) -1
- (b) 1
- (c) 0
- (d) 2

8. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left[\sqrt{x \sqrt{x + \sqrt{x}}} - \sqrt{x} \right]$$

- (a) $\frac{1}{2}$
- (b) $\frac{-1}{2}$
- (c) $\frac{1}{3}$
- (d) 1

9. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$$

- (a) -1
- (b) 0
- (c) 1
- (d) Does not exist

10. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$$

- (a) e^4
- (b) e^2
- (c) e^3
- (d) 1

11. Let $f(x)$ be a differentiable function such that $f(2) = 4$ and $f'(2) = 4$. Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$ is given by:

- (a) -4
- (b) 2
- (c) -2
- (d) 1

12. Evaluate the limit for $p > -1$:

$$\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$$

- (a) $\frac{1}{p+1}$
- (b) $\frac{1}{p-1}$
- (c) $\frac{1}{1-p}$

(d) $\frac{1}{2p+1}$

13. The value of the limit is:

$$\lim_{x \rightarrow 0} \frac{\log x^n - [x]}{[x]}, n \in \mathbb{N} \text{ where } [x] \text{ denotes greatest integer less than or equal to } x$$

- (a) Has value -1
- (b) Has value 0
- (c) Has value 1
- (d) Does not exist

14. If $f(1) = 1$ and $f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ is:

- (a) 2
- (b) 4
- (c) -1
- (d) 3

15. If $f(x+y) = f(x)f(y)$ for all x, y , and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is:

- (a) 0
- (b) 6
- (c) 1
- (d) 2

16. Evaluate the expression:

$$\lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

- (a) $\frac{1}{5}$
- (b) $\frac{1}{15}$
- (c) $\frac{1}{35}$
- (d) $\frac{2}{5}$

17. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is:

- (a) $\frac{2}{3}$
- (b) $\frac{2}{5}$
- (c) $\frac{-2}{3}$
- (d) 0

18. The value of the limit is:

$$\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$$

- (a) 1
- (b) -1
- (c) -2
- (d) 2

19. Let $f(a) = g(a) = k$ and their n th derivatives $f^{(n)}(a)$ and $g^{(n)}(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$, then the value of k is:
- (a) 4
 - (b) 1
 - (c) 2
 - (d) -1