

Detailed Solutions: Limits and Calculus Applications – Set 5

Multiple Choice Questions Solutions

1. **Question:** Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sin \left[\pi \sqrt{n^2 + 1} \right]$$

Solution: We use the identity $\sqrt{n^2 + 1} - n = \frac{(n^2 + 1) - n^2}{\sqrt{n^2 + 1} + n} = \frac{1}{\sqrt{n^2 + 1} + n}$. Thus, $\sqrt{n^2 + 1} = n + \frac{1}{\sqrt{n^2 + 1} + n}$. The argument of the sine function is:

$$\pi \sqrt{n^2 + 1} = n\pi + \frac{\pi}{\sqrt{n^2 + 1} + n}$$

Let $\theta_n = \frac{\pi}{\sqrt{n^2 + 1} + n}$. As $n \rightarrow \infty$, $\theta_n \rightarrow 0$.

$$L = \lim_{n \rightarrow \infty} \sin(n\pi + \theta_n)$$

Using $\sin(n\pi + \theta) = (-1)^n \sin(\theta)$:

$$L = \lim_{n \rightarrow \infty} (-1)^n \sin(\theta_n)$$

Since $\theta_n \rightarrow 0$, we have $\sin(\theta_n) \rightarrow 0$. The product of a bounded term $(-1)^n$ and a term tending to zero ($\sin \theta_n$) is zero.

$$L = 0$$

Answer: 0 (Option b).

2. **Question:** The value of the limit is:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} \right)$$

Solution: This is the limit of the sum of a telescoping series. The general term T_k is:

$$T_k = \frac{1}{(2k-1)(2k+1)}$$

Using partial fraction decomposition:

$$T_k = \frac{1}{2} \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right)$$

The sum of the first n terms S_n is:

$$S_n = \frac{1}{2} \sum_{k=1}^n \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right)$$
$$S_n = \frac{1}{2} \left[\left(1 - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{5} \right) + \dots + \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \right]$$

The intermediate terms cancel:

$$S_n = \frac{1}{2} \left[1 - \frac{1}{2n+1} \right]$$

The limit is:

$$L = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{2} \left[1 - \frac{1}{2n+1} \right] = \frac{1}{2} [1 - 0] = \frac{1}{2}$$

Answer: $\frac{1}{2}$ (Option a).

3. **Question:** The value of the limit is:

$$\lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}}$$

Solution: The limit is of the indeterminate form 1^∞ . We use the formula $\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x)(f(x)-1)}$.

Here $f(x) = \frac{3x-4}{3x+2}$ and $g(x) = \frac{x+1}{3}$.

$$f(x) - 1 = \frac{3x-4}{3x+2} - 1 = \frac{(3x-4) - (3x+2)}{3x+2} = \frac{-6}{3x+2}$$

The exponent limit M is:

$$M = \lim_{x \rightarrow \infty} g(x)(f(x) - 1) = \lim_{x \rightarrow \infty} \frac{x+1}{3} \cdot \frac{-6}{3x+2}$$

$$M = \lim_{x \rightarrow \infty} \frac{-2(x+1)}{3x+2}$$

Dividing numerator and denominator by x :

$$M = \lim_{x \rightarrow \infty} \frac{-2(1 + \frac{1}{x})}{3 + \frac{2}{x}} = \frac{-2(1)}{3} = -\frac{2}{3}$$

The original limit $L = e^M = e^{-2/3}$. **Answer:** $e^{-2/3}$ (Option b).

4. **Question:** Given $f'(2) = 6$ and $f'(1) = 4$, the value of the limit is:

$$\lim_{h \rightarrow 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$$

Solution: The limit is of the $\frac{0}{0}$ indeterminate form. We apply L'Hôpital's Rule.

$$L = \lim_{h \rightarrow 0} \frac{\frac{d}{dh}(f(2h+2+h^2) - f(2))}{\frac{d}{dh}(f(h-h^2+1) - f(1))}$$

Using the Chain Rule:

$$L = \lim_{h \rightarrow 0} \frac{f'(2h+2+h^2) \cdot \frac{d}{dh}(2h+2+h^2)}{f'(h-h^2+1) \cdot \frac{d}{dh}(h-h^2+1)}$$

$$L = \lim_{h \rightarrow 0} \frac{f'(2h+2+h^2) \cdot (2+2h)}{f'(h-h^2+1) \cdot (1-2h)}$$

Substitute $h = 0$:

$$L = \frac{f'(2) \cdot (2+0)}{f'(1) \cdot (1-0)} = \frac{6 \cdot 2}{4 \cdot 1} = \frac{12}{4} = 3$$

Answer: 3 (Option c).

5. **Question:** If $f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)}$, where $0 < a < \frac{1}{2}$, is:

Solution: The limit is of the $\frac{0}{0}$ form, which is the definition of the ratio of derivatives, $\frac{f'(a)}{g'(a)}$. We use the substitution $x = \tan \theta$. Since $0 < a < \frac{1}{2}$, the standard inverse trigonometric formulae apply.

- $f(x) = \cot^{-1}\left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}\right) = \cot^{-1}(\tan 3\theta).$

$$f(x) = \cot^{-1}(\cot(\frac{\pi}{2} - 3\theta)) = \frac{\pi}{2} - 3\theta = \frac{\pi}{2} - 3 \tan^{-1} x$$

$$f'(x) = \frac{d}{dx} \left(\frac{\pi}{2} - 3 \tan^{-1} x \right) = -\frac{3}{1+x^2}$$

- $g(x) = \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right) = \cos^{-1}(\cos 2\theta).$

$$g(x) = 2\theta = 2 \tan^{-1} x$$

$$g'(x) = \frac{d}{dx} (2 \tan^{-1} x) = \frac{2}{1+x^2}$$

The limit is:

$$L = \frac{f'(a)}{g'(a)} = \frac{-3/(1+a^2)}{2/(1+a^2)} = -\frac{3}{2}$$

Answer: $-\frac{3}{2}$ (Option c).

6. **Question:** If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ is:

Solution: Expand the determinant $f(x)$ along the third row for simplicity, or directly:

$$f(x) = \sin x(x^2 - x) - \cos x(x^3 - 2x^2) + \tan x(x^3 - 2x^3)$$

$$f(x) = x^2 \sin x - x \sin x - x^3 \cos x + 2x^2 \cos x - x^3 \tan x$$

Since $x \rightarrow 0$, we use the Maclaurin series expansions up to the x^2 term (since the denominator is x^2):

- $\sin x \approx x$
- $\cos x \approx 1 - \frac{x^2}{2}$
- $\tan x \approx x$

$$f(x) \approx x^2(x) - x(x) - x^3(1) + 2x^2(1 - \frac{x^2}{2}) - x^3(x) + O(x^4)$$

$$f(x) \approx x^3 - x^2 - x^3 + 2x^2 - x^4 - x^4 + O(x^4)$$

Grouping powers of x :

- x^3 terms: $x^3 - x^3 = 0$
- x^2 terms: $-x^2 + 2x^2 = x^2$

$$f(x) \approx x^2 + O(x^4)$$

The limit is:

$$L = \lim_{x \rightarrow 0} \frac{x^2 + O(x^4)}{x^2} = \lim_{x \rightarrow 0} (1 + O(x^2)) = 1$$

Answer: 1 (Option d).

7. **Question:** Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \left[x \tan x - \left(\frac{\pi}{2} \right) \sec x \right]$$

Solution: The limit is of the $\infty - \infty$ form. We rewrite the expression using $\tan x = \frac{\sin x}{\cos x}$ and $\sec x = \frac{1}{\cos x}$.

$$L = \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{x \sin x}{\cos x} - \frac{\pi/2}{\cos x} \right] = \lim_{x \rightarrow \frac{\pi}{2}} \frac{x \sin x - \pi/2}{\cos x}$$

This is of the $\frac{0}{0}$ form since $\frac{\pi}{2} \sin(\frac{\pi}{2}) - \frac{\pi}{2} = 0$. Apply L'Hôpital's Rule:

$$L = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{d}{dx}(x \sin x - \pi/2)}{\frac{d}{dx}(\cos x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 \cdot \sin x + x \cos x}{-\sin x}$$

Substitute $x = \frac{\pi}{2}$:

$$L = \frac{\sin(\frac{\pi}{2}) + \frac{\pi}{2} \cos(\frac{\pi}{2})}{-\sin(\frac{\pi}{2})} = \frac{1 + \frac{\pi}{2} \cdot 0}{-1} = \frac{1}{-1} = -1$$

Answer: -1 (Option a).

8. **Question:** Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left[\sqrt{x \sqrt{x + \sqrt{x}}} - \sqrt{x} \right]$$

Solution: The problem as stated results in an infinite limit. A common variation intended to yield one of the options is $\lim_{x \rightarrow \infty} \left[\sqrt{x + \sqrt{x}} - \sqrt{x} \right]$. We solve the intended variation.

$$L = \lim_{x \rightarrow \infty} \left[\sqrt{x + \sqrt{x}} - \sqrt{x} \right]$$

Rationalize the numerator:

$$L = \lim_{x \rightarrow \infty} \frac{(x + \sqrt{x}) - x}{\sqrt{x + \sqrt{x}} + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x}} + \sqrt{x}}$$

Divide the numerator and denominator by $\sqrt{x}$:

$$L = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x + \sqrt{x}}{x}} + 1} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{\sqrt{x}}} + 1}$$

As $x \rightarrow \infty$, $\frac{1}{\sqrt{x}} \rightarrow 0$:

$$L = \frac{1}{\sqrt{1 + 0} + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

Answer: $\frac{1}{2}$ (Option a).

9. **Question:** Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$$

Solution: We use the identity $1 - \cos 2x = 2 \sin^2 x$:

$$\sqrt{1 - \cos 2x} = \sqrt{2 \sin^2 x} = \sqrt{2} |\sin x|$$

The limit becomes:

$$L = \lim_{x \rightarrow 0} \frac{\sqrt{2} |\sin x|}{\sqrt{2}x} = \lim_{x \rightarrow 0} \frac{|\sin x|}{x}$$

Since the absolute value function is involved, we check the Left Hand Limit (LHL) and Right Hand Limit (RHL):

• **RHL** ($x \rightarrow 0^+$): For $x > 0$, $|\sin x| = \sin x$.

$$L^+ = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$$

• **LHL** ($x \rightarrow 0^-$): For $x < 0$, $|\sin x| = -\sin x$.

$$L^- = \lim_{x \rightarrow 0^-} \frac{-\sin x}{x} = -1 \cdot \lim_{x \rightarrow 0^-} \frac{\sin x}{x} = -1 \cdot 1 = -1$$

Since $L^+ \neq L^-$, the limit does not exist. **Answer:** Does not exist (Option d).

10. **Question:** Evaluate the limit:

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$$

Solution: The limit is of the 1^∞ indeterminate form. We use the formula $\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x)(f(x)-1)}$.

Here $f(x) = \frac{x^2 + 5x + 3}{x^2 + x + 3}$ and $g(x) = x$.

$$f(x) - 1 = \frac{x^2 + 5x + 3}{x^2 + x + 3} - 1 = \frac{(x^2 + 5x + 3) - (x^2 + x + 3)}{x^2 + x + 3} = \frac{4x}{x^2 + x + 3}$$

The exponent limit M is:

$$M = \lim_{x \rightarrow \infty} x \cdot \frac{4x}{x^2 + x + 3} = \lim_{x \rightarrow \infty} \frac{4x^2}{x^2 + x + 3}$$

Dividing numerator and denominator by x^2 :

$$M = \lim_{x \rightarrow \infty} \frac{4}{1 + \frac{1}{x} + \frac{3}{x^2}} = \frac{4}{1 + 0 + 0} = 4$$

The original limit $L = e^M = e^4$. **Answer:** e^4 (Option a).

11. **Question:** Let $f(x)$ be a differentiable function such that $f(2) = 4$ and $f'(2) = 4$. Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$ is given by:

Solution: The limit is of the $\frac{0}{0}$ form since the numerator is $2f(2) - 2f(2) = 0$. Apply L'Hôpital's Rule:

$$L = \lim_{x \rightarrow 2} \frac{\frac{d}{dx}(xf(2) - 2f(x))}{\frac{d}{dx}(x - 2)}$$

Since $f(2)$ is a constant:

$$L = \lim_{x \rightarrow 2} \frac{f(2) - 2f'(x)}{1}$$

Substitute $x = 2$:

$$L = f(2) - 2f'(2)$$

Using the given values $f(2) = 4$ and $f'(2) = 4$:

$$L = 4 - 2(4) = 4 - 8 = -4$$

Answer: -4 (Option a).

12. **Question:** Evaluate the limit for $p > -1$:

$$\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$$

Solution: We rewrite the expression to fit the form of a Riemann Sum:

$$L = \lim_{n \rightarrow \infty} \frac{1}{n^{p+1}} \sum_{k=1}^n k^p = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{k^p}{n^p}$$

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^p$$

This is the limit of a sum, which is equivalent to the definite integral $\int_0^1 f(x)dx$, where $f(x) = x^p$, $\frac{k}{n} = x$, and $\frac{1}{n} = dx$.

$$L = \int_0^1 x^p dx$$

Since $p > -1$, the integral is proper:

$$L = \left[\frac{x^{p+1}}{p+1} \right]_0^1 = \frac{1^{p+1}}{p+1} - \frac{0^{p+1}}{p+1} = \frac{1}{p+1}$$

Answer: $\frac{1}{p+1}$ (Option a).

13. **Question:** The value of the limit is:

$$\lim_{x \rightarrow 0} \frac{\log x^n - [x]}{[x]}, n \in \mathbb{N} \text{ where } [x] \text{ denotes greatest integer less than or equal to } x$$

Solution: We analyze the limit as x approaches 0 from the right, $x \rightarrow 0^+$, since $\log x^n$ is only defined for $x > 0$ (assuming n is any natural number). For $x \rightarrow 0^+$, x is positive and less than 1, so the greatest integer function is:

$$[x] = 0$$

The limit expression becomes $\frac{\log x^n - 0}{0}$. Since $x \rightarrow 0^+$, $\log x^n \rightarrow -\infty$.

$$L = \lim_{x \rightarrow 0^+} \frac{\log x^n}{0^+} = -\infty$$

Since the limit does not exist (it is infinite), the correct option is 'Does not exist'. **Answer:** Does not exist (Option d).

14. **Question:** If $f(1) = 1$ and $f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ is:

Solution: The limit is of the $\frac{0}{0}$ form since $\frac{\sqrt{f(1)} - 1}{\sqrt{1} - 1} = \frac{1 - 1}{1 - 1} = 0$. Apply L'Hôpital's Rule:

$$L = \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(\sqrt{f(x)} - 1)}{\frac{d}{dx}(\sqrt{x} - 1)}$$

$$L = \lim_{x \rightarrow 1} \frac{\frac{1}{2\sqrt{f(x)}} \cdot f'(x)}{\frac{1}{2\sqrt{x}}}$$

$$L = \lim_{x \rightarrow 1} \frac{f'(x)\sqrt{x}}{\sqrt{f(x)}}$$

Substitute $x = 1$:

$$L = \frac{f'(1)\sqrt{1}}{\sqrt{f(1)}} = \frac{2 \cdot 1}{\sqrt{1}} = 2$$

Answer: 2 (Option a).

15. **Question:** If $f(x + y) = f(x)f(y)$ for all x, y , and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is:

Solution: The functional equation $f(x + y) = f(x)f(y)$ implies that $f(x)$ is an exponential function (or $f(x) = 0$). First, find $f(0)$: Set $x = y = 0$: $f(0) = f(0)f(0)$. Since $f(5) = 2 \neq 0$, $f(0)$ must be 1.

Now, use the definition of the derivative for $f'(x)$:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

Using the functional equation $f(x + h) = f(x)f(h)$:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x)f(h) - f(x)}{h} = f(x) \lim_{h \rightarrow 0} \frac{f(h) - 1}{h}$$

We recognize the limit term as the definition of $f'(0)$:

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{f(h) - 1}{h}$$

We are given $f'(0) = 3$.

So, $f'(x) = f(x) \cdot 3$. To find $f'(5)$, substitute $x = 5$:

$$f'(5) = f(5) \cdot 3$$

Given $f(5) = 2$:

$$f'(5) = 2 \cdot 3 = 6$$

Answer: 6 (Option b).

16. **Question:** Evaluate the expression:

$$\lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

Solution: Let L_1 be the first limit and L_2 be the second limit.

Limit L_1 (Riemann Sum):

$$L_1 = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^4$$

This converts to the integral $\int_0^1 x^4 dx$:

$$L_1 = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5} - 0 = \frac{1}{5}$$

Limit L_2 (Degree Comparison):

$$L_2 = \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

The sum of cubes is $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4 + 2n^3 + n^2}{4}$.

$$L_2 = \lim_{n \rightarrow \infty} \frac{\frac{1}{4}n^4 + O(n^3)}{n^5}$$

Since the degree of the numerator (4) is less than the degree of the denominator (5), the limit is 0.

$$L_2 = 0$$

The final expression value is $L_1 - L_2$:

$$L = \frac{1}{5} - 0 = \frac{1}{5}$$

Answer: $\frac{1}{5}$ (Option a).

17. **Question:** If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is:

Solution: The limit is of the $\frac{0}{0}$ form since $\log(3) - \log(3) = 0$. Apply L'Hôpital's Rule:

$$k = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\log(3+x) - \log(3-x))}{\frac{d}{dx}(x)}$$

$$k = \lim_{x \rightarrow 0} \frac{\frac{1}{3+x} \cdot 1 - \frac{1}{3-x} \cdot (-1)}{1}$$

$$k = \lim_{x \rightarrow 0} \left(\frac{1}{3+x} + \frac{1}{3-x} \right)$$

Substitute $x = 0$:

$$k = \frac{1}{3+0} + \frac{1}{3-0} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Answer: $\frac{2}{3}$ (Option a).

18. **Question:** The value of the limit is:

$$\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$$

Solution: The limit is of the $\frac{0}{0}$ form. Apply L'Hôpital's Rule. For the numerator, we use the Fundamental Theorem of Calculus (Leibniz Rule): $\frac{d}{dx} \int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)$.

- Numerator derivative: $\frac{d}{dx} \int_0^{x^2} \sec^2 t dt = \sec^2(x^2) \cdot (2x) - \sec^2(0) \cdot (0) = 2x \sec^2(x^2)$.
- Denominator derivative: $\frac{d}{dx} (x \sin x) = \sin x \cdot 1 + x \cdot \cos x = \sin x + x \cos x$.

$$L = \lim_{x \rightarrow 0} \frac{2x \sec^2(x^2)}{\sin x + x \cos x}$$

This is still $\frac{0}{0}$. Divide numerator and denominator by x :

$$L = \lim_{x \rightarrow 0} \frac{2 \sec^2(x^2)}{\frac{\sin x}{x} + \cos x}$$

Now use standard limits: $\lim_{x \rightarrow 0} \sec^2(x^2) = \sec^2(0) = 1$, $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, $\lim_{x \rightarrow 0} \cos x = 1$.

$$L = \frac{2 \cdot 1^2}{1 + 1} = \frac{2}{2} = 1$$

Answer: 1 (Option a).

19. **Question:** Let $f(a) = g(a) = k$ and their n th derivatives $f^{(n)}(a)$ and $g^{(n)}(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$, then the value of k is:

Solution: First, simplify the numerator N :

$$N = f(a)g(x) - g(a)f(x)$$

Substitute the given condition $f(a) = g(a) = k$:

$$N = kg(x) - kf(x) = k(g(x) - f(x))$$

The limit expression L becomes:

$$L = \lim_{x \rightarrow a} \frac{k(g(x) - f(x))}{g(x) - f(x)}$$

The denominator approaches $g(a) - f(a) = k - k = 0$, so the form is $\frac{0}{0}$. However, since the numerator and denominator share a common factor $(g(x) - f(x))$, we can cancel this term for $x \neq a$.

$$L = \lim_{x \rightarrow a} k$$

Since k is a constant, the limit is k . We are given that the limit is 4:

$$k = 4$$

(The condition regarding the derivatives ensures that $g(x) - f(x) \neq 0$ for x in the neighbourhood of a , justifying the cancellation.) **Answer:** 4 (Option a).