SET 4

1. If $a, b, c \in \{1, 2, 3, 4, 5\}$, the number of equations of the form $ax^2 + bx + c = 0$ which have real roots is

   (a) 25

   (b) 26

   (c) 27

   (d) 24

2. In the equation $x^3 + 3Hx + G = 0$, if G and H are real and $G^2 + 4H^3 > 0$ then the roots are

   (a) all real and equal

   (b) all real and distinct

   (c) one real and two imaginary

   (d) all real and two are equal

3. If $[x]$ denotes the integral part of x and $k = \sin^{-1}\left(\frac{1+t^2}{2t}\right) > 0$ then the integral value of α for which the equation $(x - [k])(x + \alpha) - 1 = 0$ has integral roots is

   (a) 1

   (b) 2

   (c) 4

   (d) none of these

4. **True/False:** The solution of $\log_{\frac{1}{3}}(2^{x+2} - 4^x) \geq -2$ is $(-\infty, \infty)$

5. **True/False:** If $(\sqrt{2})^x + (\sqrt{3})^x = (\sqrt{13})^{x/2}$ then number of values of x is 4.

6. **True/False:** If $0 < x < 1000$ and $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x$, the number of possible values of x is 30.

7. **True/False:** If $5^x + (2\sqrt{3})^{2x} \geq 13^x$ then the solution set is $(-\infty, 2]$

8. **True/False:** If $a, b, c \in \mathbb{R}$ and $a+b+c = 0$, then the quadratic equation $4ax^2+3bx+2c = 0$ has real roots.

9. **Fill in the blank:** The polynomial $x^3 - 3x^2 - 9x + c$ can be written as $(x - \alpha)^2(x - \beta)$ if $c =$ _____

10. **Fill in the blank:** If α, β are roots of $x^2 + px - q = 0$ and γ, δ are roots of $x^2 + px + r = 0$ then $(\alpha - \gamma)(\alpha - \delta) =$ _____

11. **Fill in the blank:** If $y \neq 0$ then the number of values of the pair (x, y) such that $x + y + \frac{x}{y} = \frac{1}{2}$ and $(x + y)\frac{x}{y} = -\frac{1}{2}$ is _____

12. **Fill in the blank:** The value of α for which the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ has roots whose sum of squares is least is _____

13. **Fill in the blank:** If the equations $x^2 - 5x + 6 = 0$ and $x^2 + mx + 3 = 0$ have a common root, then $m =$ _____

14. **Multiple Correct Choice:** If $px^2 + qx + r = 0$ and $rx^2 + qx + p = 0$ have a common non-real root, then

    (a) $-2|p| < q < 2|p|$

    (b) $-2|r| < |q| < 2|r|$

    (c) $p = \pm r$

    (d) $p = r$

15. **Multiple Correct Choice:** If exactly one root of $x^2 - (t-1)x + t(t+4) = 0$ lies between the roots of $x^2 - (t+3)x + t + 2 = 0$ then

    (a) $t \in (-6, -3)$

    (b) $(3, 6)$

    (c) $t \in [-3, 1)$

    (d) $[1, 6)$