SET 4

1. Show that the area of the triangle on the argand diagram formed by the complex number z,iz and z+iz is $\frac{1}{2}|z|^2$
2. Complex numbers z_1, z_2, z_3 are the vertices A,B,C respectively of an isosceles right angled triangle with right angle at C, show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$
3. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z-z_1)}{(z-z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$
4. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$
5. $|z| \leq 1, |w| \leq 1$ show that $|z - w|^2 \leq (|z| - |w|)^2 + (\arg z - \arg w)^2$
6. Find all non zero complex numbers z satisfying $\overline{z} = iz^2$
7. Let z_1 and z_2 be roots of the equation $z^2 + pq + q =$ where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \neq 0$ and OA =OB, where O is the origin prove that $p^2 = 4q\cos^2(\frac{\alpha}{2})$
8. Let $\bar{b}z + b\overline{z} = c, b \neq 0$ be a line in the complex plane, where $\bar{b}$ is the complex conjugate of b. If a point z_1 is the reflection of the point z_2 through the line, then show that $c = \overline{z_1}b + z_2\bar{b}$
9. For complex numbers z and w, prove that $|z|^2w - |w|^2z = z - w$ if and only if z =w or $z\overline{w} = 1$
10. Let a complex number $\alpha, \alpha \neq 1$ be the root of the equation $z^{p+q} - z^p - z^q + 1 = 0$ where p,q are distinct primes, Show that either $1 + \alpha + \alpha^2 + \ldots + \alpha^{p-1}$ or $1 + \alpha + \alpha^2 + \ldots + \alpha^{q-1}$
11. If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that $|\frac{1-z_1\overline{z_2}}{z_1-z_2}| < 1$
12. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$, where $|a_r| < 2$
13. Find the centre and radius of the circle formed by all points represented by z =x+iy satisfying the relation $|\frac{z-\alpha}{z-\beta}| = k(k \neq 1)$, where α and β are constant complex numbers given by $\alpha = \alpha_1 + i\alpha_2, \beta = \beta_1 + i\beta_2$
14. If one the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}i$. Find the other vertices of the square.