Binomial Theorem - Set 3

1. The term independent of x in expansion of

$$(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}})^{10}$$

is

(a) 4
(b) 120
(c) 210
(d) 310

[Ans. c]

2. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals:

(a) 5
(b) 7
(c) 6
(d) 4

[Ans. b]

3. The sum $\sum_{i=0}^{m}(C_i^{10})(C_{m-i}^{20})$ where (C_q^p) =0 if $p>q$. is maximum when m is:

(a) 5
(b) 10
(c) 15
(d) 20

[Ans. c]

4. **ASSERTION AND REASON**

This question contains STATEMENT -1 (Assertion) and STATEMENT - II (Reason).

STATEMENT - I: $\sum_{r=0}^{n}(r+1)^nC_r=(n+2)2^{n-1}$
STATEMENT - II: $\sum_{r=0}^{n}(r+1)^nC_rx^r=(1+x)^n+nx(1+x)^{n-1}$

(a) Statement - I is True, Statement II is True. Statement II is a correct explanation of for statement I.
(b) Statement - I is True, Statement II is True. Statement II is NOT a correct explanation of for statement I.
(c) Statement - I is True, Statement II is FALSE .
(d) Statement - I is False , Statement II is TRUE.

5. If x^p occurs in the expansion of $(x^2+\frac{1}{x})^{2n}$, prove that its coefficient is

$$\frac{(2n)!}{\left(\frac{1}{3}(4n-p)\right)!\left(\frac{1}{3}(2n+p)\right)!}$$

6. Remainder when $8^{2n}-62^{2n+1}$ is divided by 9:

(a) 0

(b) 2
(c) 7
(d) 8

[Ans. 2]

7. Total number of terms in $(x+y)^{100}+(x-y)^{100}$ after simplification:

   (a) 51
   (b) 202
   (c) 100
   (d) 50

   [Ans. 51]

8. If in $(1+x)^n = 1 + a_1 x + a_2 x^2 + \cdots$, a_1, a_2, a_3 in A.P., then $n = 7$.

9. Find $\frac{a}{b}$ if in $(a-2b)^n$, 5th + 6th terms = 0. [Ans. $\frac{2(n-4)}{5}$]

10. If $|x| < 1$, coefficient of x^6 in $(1+x+x^2)^{-3}$:

   (a) 3
   (b) 6
   (c) 9
   (d) 12
   (e) 15

   [Ans. a]

11. If $(1+2x+x^2)^5 = \sum_{k=0}^{15} a_k x^k$, then $\sum_{k=0}^{7} a_{2k}$:

   (a) 128
   (b) 256
   (c) 512
   (d) 1024

   [Ans. c]

12. $49^n + 16n - 1$ is divisible by:

   (a) 3
   (b) 29
   (c) 19
   (d) 64

   [Ans. d]

13. If a_1, a_2, a_3, a_4 are consecutive binomial coefficients in $(1+x)^n$, then:

$$\frac{a_1}{a_1+a_2} + \frac{a_2}{a_3+a_4} =$$

   (a) $\frac{a_2}{a_2+a_3}$
   (b) $\frac{1}{2}\frac{a_2}{a_2+a_3}$
   (c) $\frac{2a_2}{a_2+a_3}$
   (d) $\frac{2a_3}{a_2+a_3}$

[Ans. c]

14. Greatest integer dividing $101^{100} - 1$:

    (a) 100
    (b) 1000
    (c) 10000
    (d) 100000

    [Ans. c]

15. Let n be odd. If

$$\sin^n \theta = \sum_{r=0}^{n} b_r \sin^r \theta,$$

    then:

    (a) $b_0 = 1, b_1 = 3$
    (b) $b_0 = 0, b_1 = n$
    (c) $b_0 = -1, b_1 = n$
    (d) $b_0 = 0, b_1 = n^2 - 3n + 3$

    [Ans. b]

16. If

$$\frac{1}{(1-ax)(1-bx)} = \sum_{n=0}^{\infty} a_n x^n,$$

    then

$$a_n = \frac{b^{n+1} - a^{n+1}}{b - a}.$$

17. Value of x for which 6th term of

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1})} + 1}\right]^5$$

    is 84:

    (a) 4
    (b) 3
    (c) 2
    (d) 5

    [Ans. c]