SET 3

1. If w=$\alpha + i\beta$, where $\beta \neq 0$, satisfies the condition that $(\frac{w-\overline{w}z}{1-z})$ is purely real, then the set of values of z is : A. $|z| = 1, z \neq 2$ B. $|z| = 1$ and $z \neq 1$ C. z $=\overline{z}$ D. none of these

2. A man walks a distance of 3 units from the origin towards the north east $(N45°E)$ direction. From there, he walks a distance of 4 units towards the north west $(N45°W)$ direction to reach a point P. Then the position of P in the Argand plane is : A. $3e^{\frac{i\pi}{4}} + 4i$ B. $(3-4i)e^{\frac{i\pi}{4}}$ C. $(4+3i)e^{\frac{i\pi}{4}}$ D. $(3+4i)e^{\frac{i\pi}{4}}$

3. If $|z| = 1$ and z $\neq \pm 1$ then the values of $\frac{z}{1-z^2}$ lie on : A. a line not passing through the origin B. $|z| = \sqrt{2}$ C. x axis D. y axis

Fill in the Blanks

4. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. If the points D and M represent the complex numbers 1+i and 2-i respectively, then A represents the complex number ...... or .....

5. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$ If $z_1 = 1 + i\sqrt{3}$, then $z_2 =$ ...., $z_3 =$ .....

6. The value of the expression $(2-\omega)(2-\omega^2) + 2(3-\omega)(3-\omega^2) + ... + (n-1).(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity, is....

TRUE / FALSE

7. The cube roots of unity when represented on argand diagram form the vertices of an equilateral triangle.

OBJECTIVE QUESTIONS More than one options are correct

8. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $Re(z_1\overline{z_2})$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies : A. $|w_1| = 1$ B. $|w_2| = 1$ C. $Re(w_1\overline{w_2}) = 0$ D. none of these

9. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$ if z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be : A. zero B. real and positive C. real and negative D. purely imaginary E. none of these

SUBJECTIVE QUESTIONS

10. It is given that n is an odd integer greater than 3, but n is not a multiple of 3. Prove that $x^3 + x^2 + x$ is a factor of $(x+1)^n - x^n - 1$.

11. Find the real values of x and y for which the following equation is satisfied : $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

12. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumference of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$

13. A relation R on the set of complex numbers is defined by $z_1 R z_2$, if and only if $\frac{z_1-z_2}{z_1+z_2}$ is real. Show that R is an equivalence relation.

14. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1z_2 = 0$

15. If $1, a_1, z_2, \ldots, z_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-z_2)(1-a_3)\ldots(1-a_{n-1}) = n$