SETS, RELATIONS & FUNCTIONS - SET 5

1. If A = $\{(x, y) : x^2 + y^2 = 25\}$ and B = $\{(x, y) : x^2 + 9y^2 = 144\}$ then $A \cap B$ contains
   (a) one point
   (b) three points
   (c) two points
   (d) four points
2. If $f : R \to R$ satisfies f(x+y) = f(x) +f(y), for all $x, y \in R$ and f(1)=7, then $\sum_{r=1}^{n}$ f(r) is
   (a) $\frac{7n}{2}$
   (b) $\frac{7(n+1)}{2}$
   (c) $7n(n + 1)$
   (d) $\frac{7n(n+1)}{2}$
3. The function f(x) = log(x+$\sqrt{x^2 + 1}$) is
   (a) an even function
   (b) an odd function
   (c) a periodic function
   (d) neither an even nor an odd function
4. Let R = $\{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set A = $\{1, 2, 3, 4\}$. The relation R is :
   (a) function
   (b) transitive
   (c) not symmetric
   (d) reflexive
5. Let f: $(-1, 1) \to B$, be a function defined by f(x) = $\tan^{-1} \frac{2x}{1-x^2}$, then f is both one one and onto when B is the interval :
   (a) $(\frac{-\pi}{2}, \frac{\pi}{2})$
   (b) $[\frac{-\pi}{2}, \frac{\pi}{2}]$
   (c) $[0, \frac{\pi}{2})$
   (d) $(0, \frac{\pi}{2})$
6. Let W denote the words in the English dictionary. Define the relation R by : R = $\{(x, y) \in W \times W|$ the words x and y have at least one letter in common$\}$. Then R is :
   (a) reflexive, symmetric and not transitive
   (b) reflexive, symmetric and transitive
   (c) reflexive, not symmetric and transitive
   (d) not reflexive, symmetric and transitive
7. If f(x) =$\sqrt{x}$, g(x) =$\frac{x}{4}$ and h(x) =4x-8 then
   (a) gohof(x) = $\sqrt{x - 2}$
   (b) fogoh(x) = $\sqrt{x} - 2$

(c) hogof(x) = $\sqrt{x} - 8$

(d) hofog(x) = $\sqrt{x} - 4$

8. Which of the following sets of ordered pairs define a one to one function ?

   (a) R = {(x,y); $x^2 + y^2 = 2$} on R.

   (b) A = {1,2,3}, B ={1,2,3,4,5} and R = {(x,y): 5x+2y is a prime number} on A

   (c) A ={1,2,3,4}, B={1,2,3,4,5,6,7} and R={(x,y):y=$x^2 - 3x + 3$} on A.

   (d) none of these

9. Let f: R $\rightarrow$ R be a function defined by f(x) =$\frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then

   (a) f is both one one and onto

   (b) f is one one but not onto

   (c) f is onto but not one one

   (d) f is neither one one nor onto

10. If f(x) is a polynomial satisfying f(x) f($\frac{1}{x}$), and f(3)=28, then f(4) is given by

    (a) 63

    (b) 65

    (c) 67

    (d) 68

11. Let the function f(x) = $x^2 + x + sinx - cosx + log(1 + |x|)$ be defined on the interval [0,1]. The function g(x) on [-1,1] satisfying g(-x) = -f(x) is

    (a) $x^2 + x + sinx + cosx - log(1 + |x|)$

    (b) $-x^2 + x + sinx + cosx - log(1 + |x|)$

    (c) $-x^2 + x + sinx - cosx - log(1 + |x|)$

    (d) none of these

12. Let f(x) =$a^x (a > 0)$ be written as f(x) =g(x)+h(x), where g(x) is an even function and h(x) is an odd function. Then the value of g(x+y)+g(x-y) is

    (a) 2g(x)g(y)

    (b) 2g(x+y)g(x-y)

    (c) 2g(x)

    (d) none of these

13. Let f(x) =arcsin(sinx). Then

    (a) f is periodic with period π

    (b) f is periodic with period $\frac{\pi}{2}$

    (c) f is periodic with period 2π

    (d) f is non periodic

14. Let a function f: R $\rightarrow$ R be defined by f(x) =2x+sinx for $x \in R$. Then f is

    (a) one to one and onto

(b) one to one not onto

(c) onto but not one to one

(d) neither one to one nor onto

15. Suppose f(x) $=(x+1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of f(x) w.r.t y=x then g(x) equals

(a) $-\sqrt{x}-1, x \geq 0$

(b) $\frac{1}{(x+1)^2}, x > -1$

(c) $\sqrt{x+1}, x \geq -1$

(d) $\sqrt{x}-1, x \geq 0$

16. Let n(A) =4, n(B) =6. Then the number of one one functions from A to B is

(a) 120

(b) 360

(c) 24

(d) none of these

17. The graph of the function f(x) = $log_a(x+\sqrt{x^2+1})$ is symmetric about

(a) x axis

(b) origin

(c) y axis

(d) the line y =x

SET 5 - ADVANCED FUNCTION PROBLEMS

1. **For problems 1 - 3:** Let f: $N \to R$ be a function satisfying the following conditions: $f(1) = \frac{1}{2}$ and f(1) + 2f(2) + 3f(3) + $\cdots$ + nf(n) = n(n+1)f(n) for $n \geq 2$

2. The value of $f(1003) = \frac{1}{K}$, where K equals

(a) 2006

(b) 2003

(c) 1003

(d) 2005

3. The value of $f(999)$ is $\frac{1}{K}$ where K is

(a) 1998

(b) 999

(c) 1000

(d) 2000

4. f(1), f(2), f(3), f(4) $\cdots$ represents a series of

(a) an A.P

(b) a G.P

(c) a H.P

(d) none of these

5. Let function f: R$\rightarrow R$ be defined by f(x) =2x+sinx for $x \in R$. Then f is :

   (a) one to one and onto
   (b) one to one but not onto
   (c) onto but not one to one
   (d) neither one to one nor onto

6. If f: $[0, \infty) \rightarrow [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then f is :

   (a) one one and onto
   (b) one one but not onto
   (c) onto but not one one
   (d) neither one one nor onto

7. Let $f : R \rightarrow R$ be a continuous function satisfying f(x) + f(-x) = 0, $\forall x \in R$. If $f(-3) = 2$ and $f(5) = 4$ in [-5,5] then minimum number of roots of the equation f(x) =0 is .....

8. If $f(x) = sin^2x + sin^2(x + \frac{\pi}{3}) + cosxcos(x + \frac{\pi}{3})$ and g($\frac{5}{4}$)=1, then (gof)(x) =.....

9. The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$ is

   (a) $0 < x \leq 1$
   (b) $0 \leq x \leq 1$
   (c) $-\infty < x \leq 0$
   (d) $-\infty < x < 1$