Self Assessment Test - 1

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Time: 1.5 Hours — M.M.: 60

Class: 9 Standard
Chapter: Surface Areas and Volumes
Topics: Surface areas and volumes of cubes, cuboids, spheres, hemispheres, right circular cylinders/cones; conversion of units.

Section A — Multiple Choice Questions ($8 \times 1 = 8$ Marks)

1. A solid metallic sphere of radius 10 cm is melted and recast into small cones, each of radius 2 cm and height 4 cm. The number of cones recast is:
   (a) 125
   (b) 250
   (c) 375
   (d) 500
2. The total surface area of a solid hemisphere is 462 cm^2. Its diameter is:
   (a) 7 cm
   (b) 14 cm
   (c) 21 cm
   (d) 28 cm
3. If the ratio of the volumes of two cubes is 8 : 27, then the ratio of their total surface areas is:
   (a) 2 : 3
   (b) 4 : 9
   (c) 8 : 27
   (d) 1 : 3
4. A cylindrical pencil is sharpened to a conical point. The volume of the material removed is what fraction of the whole pencil's volume (assuming the cone's base is the same as the cylinder's and height of the removed part is h)?
   (a) $\frac{1}{3}$
   (b) $\frac{2}{3}$
   (c) $\frac{1}{2}$
   (d) Information insufficient

5. The edges of a cuboid are in the ratio 1 : 2 : 3 and its total surface area is 88 cm^2. The volume of the cuboid is:
   (a) 48 cm^3
   (b) 24 cm^3
   (c) 64 cm^3
   (d) 12 cm^3
6. How many litres of water can a hemispherical tank hold if its inner radius is 3.5 m? ($\pi = 22/7$)
   (a) 89.83 litres
   (b) 89833 litres
   (c) 89833.33 litres
   (d) 898333.33 litres
7. The length of the longest rod that can be placed in a room 12 m long, 9 m wide, and 8 m high is:
   (a) 17 m
   (b) 16 m
   (c) 15 m
   (d) 14 m

Section B — Short Answer Questions ($6 \times 2 = 12$ Marks)

8. A cube of side 4 cm is painted on all its faces and then cut into smaller cubes of side 1 cm. Find the number of smaller cubes that have exactly two painted faces.
9. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to level a playground. Find the area of the playground in m^2 ($\pi = 22/7$).
10. Two solid right circular cones of equal height and radii r_1 and r_2 are melted and reformed into a single solid right circular cone of equal height h'. If $h = h'$, show that $r'^3 = r_1^3 + r_2^3$ is false and find the correct relation.
11. The curved surface area of a cylinder is 1320 cm^2 and its base radius is 10.5 cm. Find its volume ($\pi = 22/7$).
12. A hemispherical bowl is made of 0.25 cm thick steel. The inner radius is 5 cm. Find the volume of steel used ($\pi = 3.14$).
13. A rectangular sheet of paper 44 cm $\times$ 18 cm is rolled along its length to form a cylinder. Find the volume ($\pi = 22/7$).

Section C — Long Answer Questions ($4 \times 4 = 16$ Marks)

14. A tent is in the shape of a right circular cylinder up to height 3 m and conical above it. The diameter of the base is 10.5 m and the slant height of the conical part is 5 m. Find the total cost of canvas required if canvas costs Rs. 10 per m^2 ($\pi = 22/7$).

15. A river 3 m deep and 40 m wide flows at 2 km/hr. How much water falls into the sea in a minute (in m^3)?

16. A storage tank is composed of a cylinder of radius r and height h with a hemisphere on top. If total height $H = r + h$, express the total surface area in terms of r and H.

17. The inner diameter of a glass is 7 cm and height 16 cm. The bottom has a hemispherical raised portion. Find its capacity ($\pi = 22/7$).

Section D — Case Study Based Question ($1 \times 5 = 25$ Marks)

Case Study: A group of mountaineers decided to set up a base camp in a hilly region. They planned to construct a dome-shaped shelter for accommodation. The shelter is designed in the shape of a hemisphere (dome) with a metallic frame. Its volume is 179.67 m^3. The cost of insulation for the inner surface is Rs. 50 per m^2, and reflective paint for the outer surface costs Rs. 20 per m^2. ($\pi = \frac{22}{7}$)

18. The volume of a hemisphere is given by $V = \frac{2}{3}\pi r^3$. If $V = 179.67$ m^3, the radius r of the dome is approximately:

    (a) 3.5 m

    (b) 3.6 m

    (c) 3.7 m

    (d) 4.0 m

19. The inner surface area of the hemisphere that needs to be insulated is:

$$A = 2\pi r^2$$

    The area (in m^2) is approximately:

    (a) 80 m^2

    (b) 82 m^2

    (c) 90 m^2

    (d) 100 m^2

20. The total cost of applying reflective paint to the outer surface of the dome is:

(a) Rs. 1640

(b) Rs. 2000

(c) Rs. 2100

(d) Rs. 2400

21. The total cost of insulating the inner surface of the dome is:

(a) Rs. 4100

(b) Rs. 4200

(c) Rs. 5000

(d) Rs. 6000

22. If the circular floor of the dome (base) also needs to be covered with a mat costing Rs. 100 per m^2, then the cost of the mat will be:

$$\text{Area of floor} = \pi r^2$$

The total cost is approximately:

(a) Rs. 4000

(b) Rs. 4200

(c) Rs. 4600

(d) Rs. 5000

Answers and Detailed Solutions

Section A - Multiple Choice Questions

1. **Answer:** (b) 250 **Solution:** Let R be the radius of the sphere and r and h be the radius and height of the cone. $R = 10$ cm, $r = 2$ cm, $h = 4$ cm. Volume of sphere $V_{\text{sphere}} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi$ cm^3. Volume of one cone $V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2)^2(4) = \frac{16}{3}\pi$ cm^3. Number of cones $n = \frac{V_{\text{sphere}}}{V_{\text{cone}}} = \frac{\frac{4000}{3}\pi}{\frac{16}{3}\pi} = \frac{4000}{16} = 250$.

2. **Answer:** (b) 14 cm **Solution:** Total Surface Area (TSA) of a solid hemisphere $= 3\pi r^2$. Given $3\pi r^2 = 462$ cm^2. $3 \times \frac{22}{7} \times r^2 = 462$. $r^2 = \frac{462\times7}{3\times22} = \frac{462\times7}{66}$. $r^2 = 7 \times 7 = 49$. $r = 7$ cm. Diameter $d = 2r = 2 \times 7 = 14$ cm.

3. **Answer:** (b) 4 : 9 **Solution:** Let a_1 and a_2 be the edges of the two cubes. Ratio of volumes $\frac{V_1}{V_2} = \frac{a_1^3}{a_2^3} = \frac{8}{27} = \left(\frac{2}{3}\right)^3$. Ratio of edges $\frac{a_1}{a_2} = \frac{2}{3}$. Total Surface Area (TSA) of a cube $= 6a^2$. Ratio of TSAs $\frac{\text{TSA}_1}{\text{TSA}_2} = \frac{6a_1^2}{6a_2^2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

4. **Answer:** (b) 2/3 **Solution:** Let r be the radius of the cylinder/cone and h be the height of the removed part (cone). Volume of the cylindrical part removed (if it were not sharpened) $V_{\text{cyl}} = \pi r^2 h$. Volume of the conical part (pencil tip) $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$. Volume of material removed $V_{\text{removed}} = V_{\text{cyl}} - V_{\text{cone}} = \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$. The volume removed is $\frac{2}{3}$ of the volume of the cylindrical part corresponding to the removed section, V_{cyl}. The question asks for the fraction of the *whole pencil's volume*. Assuming the *whole pencil's volume* refers to the *volume of the cylindrical part that was sharpened* (as the volume of the whole pencil is unknown), the fraction is: Fraction $= \frac{V_{\text{removed}}}{V_{\text{cyl}}} = \frac{\frac{2}{3}\pi r^2 h}{\pi r^2 h} = \frac{2}{3}$.

5. **Answer:** (a) 48 cm^3 **Solution:** Let the edges be $l = x$, $b = 2x$, $h = 3x$. TSA of cuboid $= 2(lb + bh + hl) = 88$ cm^2. $2(x(2x) + 2x(3x) + 3x(x)) = 88$. $2(2x^2 + 6x^2 + 3x^2) = 88$. $2(11x^2) = 88 \implies 22x^2 = 88$. $x^2 = 4 \implies x = 2$ (since length must be positive). The edges are $l = 2$ cm, $b = 4$ cm, $h = 6$ cm. Volume $V = lbh = 2 \times 4 \times 6 = 48$ cm^3.

6. **Answer:** (d) 898333.33 litres **Solution:** Inner radius $r = 3.5$ m. Volume of hemispherical tank $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$. $V = \frac{2}{3} \times \frac{22}{7} \times 42.875 = \frac{44}{21} \times 42.875 \approx 89.8333$ m^3. Since 1 m^3 = 1000 litres, Capacity in litres $= 89.8333 \times 1000 \approx 898333.33$ litres.

7. **Answer:** (a) 17 m **Solution:** Length of the longest rod (diagonal of the cuboid) $D = \sqrt{l^2 + b^2 + h^2}$. $D = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$ m.

Section B - Short Answer Questions

8. **Solution:** A cube of side 4 cm is cut into $(4/1)^3 = 64$ smaller cubes. Cubes with exactly two painted faces are located on the edges, excluding the corners. Number of small cubes on each edge $= 4/1 = 4$. Number of small cubes with two painted faces on one edge $= 4 - 2 = 2$ (excluding the two corner cubes). Total number of edges in a cube $= 12$. Total number of smaller cubes with exactly two painted faces $= 12 \times 2 = 24$.

9. **Solution:** Diameter of roller $d = 84$ cm $\implies r = 42$ cm $= 0.42$ m. Length of roller $h = 120$ cm $= 1.2$ m. Area covered in one revolution = Curved Surface Area (CSA) of the cylinder $= 2\pi rh$. CSA $= 2 \times \frac{22}{7} \times 0.42 \times 1.2 = 2 \times 22 \times 0.06 \times 1.2 = 3.168$ m^2. Area of the playground = Area covered in 500 revolutions. Area $= 500 \times 3.168 = 1584$ m^2.

10. **Solution:** Let V_1 and V_2 be the volumes of the two cones. Let V' be the volume of the single cone. $V_1 = \frac{1}{3}\pi r_1^2 h$, $V_2 = \frac{1}{3}\pi r_2^2 h$. $V' = \frac{1}{3}\pi r'^2 h'$. By conservation of volume: $V' = V_1 + V_2$. $\frac{1}{3}\pi r'^2 h' = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$. $\frac{1}{3}\pi r'^2 h' = \frac{1}{3}\pi h(r_1^2 + r_2^2)$. Given $h = h'$, we can cancel $\frac{1}{3}\pi h$ from both sides. $r'^2 = r_1^2 + r_2^2$. The statement $r'^3 = r_1^3 + r_2^3$ is **false**. The correct relation is $r'^2 = r_1^2 + r_2^2$.

11. **Solution:** Curved Surface Area (CSA) of cylinder $= 2\pi rh = 1320$ cm^2. Base radius $r = 10.5$ cm. $2 \times \frac{22}{7} \times 10.5 \times h = 1320$. $44 \times 1.5 \times h = 1320$. $66h = 1320 \implies h = \frac{1320}{66} = 20$ cm. Volume of the cylinder $V = \pi r^2 h = \frac{22}{7} \times (10.5)^2 \times 20$. $V = \frac{22}{7} \times 110.25 \times 20 = 22 \times 15.75 \times 20 = 6930$ cm^3.

12. **Solution:** Inner radius $r_1 = 5$ cm. Thickness $t = 0.25$ cm. Outer radius $r_2 = r_1 + t = 5 + 0.25 = 5.25$ cm. Volume of steel used V_{steel} = Volume of outer hemisphere − Volume of inner hemisphere. $V_{\text{steel}} = \frac{2}{3}\pi r_2^3 - \frac{2}{3}\pi r_1^3 = \frac{2}{3}\pi(r_2^3 - r_1^3)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times ((5.25)^3 - (5)^3)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times (144.703125 - 125)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times 19.703125 \approx 41.28$ cm^3.

13. **Solution:** When the rectangular sheet 44 cm $\times$ 18 cm is rolled along its length (44 cm), the length becomes the circumference of the cylinder's base, and the width becomes its height. Circumference $C = 2\pi r = 44$ cm. $2 \times \frac{22}{7} \times r = 44 \implies r = 7$ cm. Height of the cylinder $h = 18$ cm. Volume of the cylinder $V = \pi r^2 h = \frac{22}{7} \times (7)^2 \times 18$. $V = 22 \times 7 \times 18 = 154 \times 18 = 2772$ cm^3.

Section C - Long Answer Questions

14. **Solution:** The tent consists of a cylinder and a cone. Canvas is required for the CSA of both. **Cylindrical part:** Height $h_{\text{cyl}} = 3$ m. Diameter $D = 105$ m $\implies$ radius $r = 52.5$ m. CSA$_{\text{cyl}} = 2\pi r h_{\text{cyl}} = 2 \times \frac{22}{7} \times 52.5 \times 3$. CSA$_{\text{cyl}} = 2 \times 22 \times 7.5 \times 3 = 990$ m^2. **Conical part:** Radius $r = 52.5$ m. Slant height $l = 50$ m. CSA$_{\text{cone}} = \pi rl = \frac{22}{7} \times 52.5 \times 50$. CSA$_{\text{cone}} = 22 \times 7.5 \times 50 = 8250$ m^2. **Total Area and Cost:** Total area of canvas = CSA$_{\text{cyl}}$ + CSA$_{\text{cone}}$ $= 990 + 8250 = 9240$ m^2. Total cost = Total Area $\times$ Cost per m^2 $= 9240 \times 10 =$ Rs. 92400.

15. **Solution:** The water flowing in a minute forms a cuboid. Depth $h = 3$ m, Width $b = 40$ m. Rate of flow (length of the cuboid formed per hour) $L = 2$ km/hr $=$ 2000 m/hr. Length of water falling into the sea in 1 minute $l = L \times \frac{1}{60}$ hour/min. $l = 2000 \times \frac{1}{60} = \frac{200}{6} = \frac{100}{3}$ m. Volume of water falling into the sea per minute $V = l \times b \times h$. $V = \frac{100}{3} \times 40 \times 3 = 100 \times 40 = 4000$ m^3.

16. **Solution:** The tank consists of a cylinder with radius r and height h and a hemisphere with radius r. Total height of the tank $H = r + h$. Therefore, $h = H - r$. Total Surface Area (TSA) of the tank: TSA = Base Area (circular) + CSA of cylinder + CSA of hemisphere. Since the base is the ground, and the cylinder is joined to the hemisphere (internal joint area is not counted), the TSA is: TSA = Area of the circular base + CSA of cylinder + CSA of hemisphere. TSA =

$\pi r^2 + 2\pi rh + 2\pi r^2$. TSA $= 3\pi r^2 + 2\pi rh$. Substitute $h = H - r$ into the equation: TSA $= 3\pi r^2 + 2\pi r(H - r)$. TSA $= 3\pi r^2 + 2\pi rH - 2\pi r^2$. TSA $= \pi r^2 + 2\pi rH = \pi r(r + 2H)$.

17. **Solution:** The actual capacity of the glass is the volume of the cylindrical glass minus the volume of the hemispherical raised portion. **Cylindrical part:** Inner diameter $D = 7$ cm $\implies$ radius $R = 3.5$ cm. Height $H = 16$ cm. Volume of the cylinder $V_{\text{cyl}} = \pi R^2 H = \frac{22}{7} \times (3.5)^2 \times 16$. $V_{\text{cyl}} = \frac{22}{7} \times 12.25 \times 16 = 22 \times 1.75 \times 16 = 616$ cm^3. **Hemispherical part:** Radius of the hemisphere $r = 3.5$ cm (same as the cylinder's base radius). Volume of the hemisphere $V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$. $V_{\text{hemi}} = \frac{2}{3} \times \frac{22}{7} \times 42.875 \approx 89.83$ cm^3. $V_{\text{hemi}} = \frac{1}{3} \times (\pi r^2) \times (2r) = \frac{1}{3} \times \left(\frac{22}{7} \times (3.5)^2\right) \times 7 = \frac{1}{3} \times 38.5 \times 7 = \frac{269.5}{3} \approx 89.833$ cm^3. **Actual Capacity:** Actual Capacity $V_{\text{act}} = V_{\text{cyl}} - V_{\text{hemi}} = 616 - 89.833 = 526.167$ cm^3.

Section D — Case Study Solutions

Given: Volume of the hemispherical dome, $V = 179.67\text{ m}^3$ Cost of insulation for inner surface = Rs. 50 per m^2 Cost of reflective paint for outer surface = Rs. 20 per m^2 $\pi = \frac{22}{7}$

Formulae Used:

- Volume of a hemisphere: $V = \frac{2}{3}\pi r^3$
- Curved surface area (C.S.A.) of a hemisphere: $A = 2\pi r^2$
- Area of circular base: $A_{\text{base}} = \pi r^2$

Q1. Finding the radius of the dome

$$V = \frac{2}{3}\pi r^3 = 179.67$$

Substitute $\pi = \frac{22}{7}$:

$$\frac{2}{3} \times \frac{22}{7} \times r^3 = 179.67$$

$$r^3 = \frac{179.67 \times 3 \times 7}{2 \times 22} = \frac{3773.07}{44} \approx 85.75$$

$$r = \sqrt[3]{85.75} \approx 4.4\text{ m}$$

Answer: The radius of the dome is approximately $\boxed{4.4\text{ m}}$.

Q2. Finding the inner surface area (to be insulated)

$$A = 2\pi r^2$$

$$A = 2 \times \frac{22}{7} \times (4.4)^2 = \frac{44}{7} \times 19.36 = 121.7\text{ m}^2$$

Answer: The inner surface area is approximately $\boxed{122\text{ m}^2}$.

Q3. Cost of reflective paint on the outer surface

$$\text{Cost} = \text{Area} \times \text{Rate}$$

$$\text{Cost} = 122 \times 20 = 2440$$

Answer: The cost of reflective paint is $\boxed{\text{Rs. } 2440}$.

Q4. Cost of insulation on the inner surface

$$\text{Cost} = 122 \times 50 = 6100$$

Answer: The cost of insulation is $\boxed{\text{Rs. } 6100}$.

Q5. Cost of mat for the circular floor

Area of the floor:

$$A_{\text{base}} = \pi r^2 = \frac{22}{7} \times (4.4)^2 = 60.8 \text{ m}^2$$

Cost of mat:

$$\text{Cost} = 60.8 \times 100 = 6080$$

Answer: The cost of mat to cover the floor is $\boxed{\text{Rs. } 6080}$.

Final Summary:

Q.No.	Description	Answer
1	Radius of dome	4.4 m
2	Inner surface area	122 m^2
3	Cost of reflective paint	Rs. 2440
4	Cost of insulation	Rs. 6100
5	Cost of mat for floor	Rs. 6080