# Answers and Detailed Solutions

## Section A - Multiple Choice Questions

1. **Answer:** (b) 250 **Solution:** Let $R$ be the radius of the sphere and $r$ and $h$ be the radius and height of the cone. $R = 10$ cm, $r = 2$ cm, $h = 4$ cm. Volume of sphere $V_{\text{sphere}} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi$ cm$^3$. Volume of one cone $V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2)^2(4) = \frac{16}{3}\pi$ cm$^3$. Number of cones $n = \frac{V_{\text{sphere}}}{V_{\text{cone}}} = \frac{\frac{4000}{3}\pi}{\frac{16}{3}\pi} = \frac{4000}{16} = 250$.

2. **Answer:** (b) 14 cm **Solution:** Total Surface Area (TSA) of a solid hemisphere $= 3\pi r^2$. Given $3\pi r^2 = 462$ cm$^2$. $3 \times \frac{22}{7} \times r^2 = 462$. $r^2 = \frac{462\times 7}{3\times 22} = \frac{462\times 7}{66}$. $r^2 = 7 \times 7 = 49$. $r = 7$ cm. Diameter $d = 2r = 2 \times 7 = 14$ cm.

3. **Answer:** (b) 4 : 9 **Solution:** Let $a_1$ and $a_2$ be the edges of the two cubes. Ratio of volumes $\frac{V_1}{V_2} = \frac{a_1^3}{a_2^3} = \frac{8}{27} = \left(\frac{2}{3}\right)^3$. Ratio of edges $\frac{a_1}{a_2} = \frac{2}{3}$. Total Surface Area (TSA) of a cube $= 6a^2$. Ratio of TSAs $\frac{\text{TSA}_1}{\text{TSA}_2} = \frac{6a_1^2}{6a_2^2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

4. **Answer:** (b) 2/3 **Solution:** Let $r$ be the radius of the cylinder/cone and $h$ be the height of the removed part (cone). Volume of the cylindrical part removed (if it were not sharpened) $V_{\text{cyl}} = \pi r^2 h$. Volume of the conical part (pencil tip) $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$. Volume of material removed $V_{\text{removed}} = V_{\text{cyl}} - V_{\text{cone}} = \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$. The volume removed is $\frac{2}{3}$ of the volume of the cylindrical part corresponding to the removed section, $V_{\text{cyl}}$. The question asks for the fraction of the *whole pencil's volume*. Assuming the *whole pencil's volume* refers to the *volume of the cylindrical part that was sharpened* (as the volume of the whole pencil is unknown), the fraction is: Fraction $= \frac{V_{\text{removed}}}{V_{\text{cyl}}} = \frac{\frac{2}{3}\pi r^2 h}{\pi r^2 h} = \frac{2}{3}$.

5. **Answer:** (a) 48 cm$^3$ **Solution:** Let the edges be $l = x$, $b = 2x$, $h = 3x$. TSA of cuboid $= 2(lb + bh + hl) = 88$ cm$^2$. $2(x(2x) + 2x(3x) + 3x(x)) = 88$. $2(2x^2 + 6x^2 + 3x^2) = 88$. $2(11x^2) = 88 \implies 22x^2 = 88$. $x^2 = 4 \implies x = 2$ (since length must be positive). The edges are $l = 2$ cm, $b = 4$ cm, $h = 6$ cm. Volume $V = lbh = 2 \times 4 \times 6 = 48$ cm$^3$.

6. **Answer:** (d) 898333.33 litres **Solution:** Inner radius $r = 3.5$ m. Volume of hemispherical tank $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$. $V = \frac{2}{3} \times \frac{22}{7} \times 42.875 = \frac{44}{21} \times 42.875 \approx 89.8333$ m$^3$. Since 1 m$^3$ = 1000 litres, Capacity in litres $= 89.8333 \times 1000 \approx 898333.33$ litres.

7. **Answer:** (a) 17 m **Solution:** Length of the longest rod (diagonal of the cuboid) $D = \sqrt{l^2 + b^2 + h^2}$. $D = \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$ m.

## Section B - Short Answer Questions

8. **Solution:** A cube of side 4 cm is cut into $(4/1)^3 = 64$ smaller cubes. Cubes with exactly two painted faces are located on the edges, excluding the corners. Number of small cubes on each edge $= 4/1 = 4$. Number of small cubes with two painted faces on one edge $= 4 - 2 = 2$ (excluding the two corner cubes). Total number of edges in a cube $= 12$. Total number of smaller cubes with exactly two painted faces $= 12 \times 2 = 24$.

9. **Solution:** Diameter of roller $d = 84$ cm $\implies r = 42$ cm $= 0.42$ m. Length of roller $h = 120$ cm $= 1.2$ m. Area covered in one revolution = Curved Surface Area (CSA) of the cylinder $= 2\pi rh$. CSA $= 2 \times \frac{22}{7} \times 0.42 \times 1.2 = 2 \times 22 \times 0.06 \times 1.2 = 3.168$ m$^2$. Area of the playground = Area covered in 500 revolutions. Area $= 500 \times 3.168 = 1584$ m$^2$.

10. **Solution:** Let $V_1$ and $V_2$ be the volumes of the two cones. Let $V'$ be the volume of the single cone. $V_1 = \frac{1}{3}\pi r_1^2 h$, $V_2 = \frac{1}{3}\pi r_2^2 h$. $V' = \frac{1}{3}\pi r'^2 h'$. By conservation of volume: $V' = V_1 + V_2$. $\frac{1}{3}\pi r'^2 h' = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$. $\frac{1}{3}\pi r'^2 h' = \frac{1}{3}\pi h(r_1^2 + r_2^2)$. Given $h = h'$, we can cancel $\frac{1}{3}\pi h$ from both sides. $r'^2 = r_1^2 + r_2^2$. The statement $r'^3 = r_1^3 + r_2^3$ is **false**. The correct relation is $r'^2 = r_1^2 + r_2^2$.

11. **Solution:** Curved Surface Area (CSA) of cylinder $= 2\pi rh = 1320$ cm$^2$. Base radius $r = 10.5$ cm. $2 \times \frac{22}{7} \times 10.5 \times h = 1320$. $44 \times 1.5 \times h = 1320$. $66h = 1320 \implies h = \frac{1320}{66} = 20$ cm. Volume of the cylinder $V = \pi r^2 h = \frac{22}{7} \times (10.5)^2 \times 20$. $V = \frac{22}{7} \times 110.25 \times 20 = 22 \times 15.75 \times 20 = 6930$ cm$^3$.

12. **Solution:** Inner radius $r_1 = 5$ cm. Thickness $t = 0.25$ cm. Outer radius $r_2 = r_1 + t = 5 + 0.25 = 5.25$ cm. Volume of steel used $V_{\text{steel}}$ = Volume of outer hemisphere $-$ Volume of inner hemisphere. $V_{\text{steel}} = \frac{2}{3}\pi r_2^3 - \frac{2}{3}\pi r_1^3 = \frac{2}{3}\pi(r_2^3 - r_1^3)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times ((5.25)^3 - (5)^3)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times (144.703125 - 125)$. $V_{\text{steel}} = \frac{2}{3} \times 3.14 \times 19.703125 \approx 41.28$ cm$^3$.

13. **Solution:** When the rectangular sheet 44 cm $\times$ 18 cm is rolled along its length (44 cm), the length becomes the circumference of the cylinder's base, and the width becomes its height. Circumference $C = 2\pi r = 44$ cm. $2 \times \frac{22}{7} \times r = 44 \implies r = 7$ cm. Height of the cylinder $h = 18$ cm. Volume of the cylinder $V = \pi r^2 h = \frac{22}{7} \times (7)^2 \times 18$. $V = 22 \times 7 \times 18 = 154 \times 18 = 2772$ cm$^3$.

## Section C - Long Answer Questions

14. **Solution:** The tent consists of a cylinder and a cone. Canvas is required for the CSA of both. **Cylindrical part:** Height $h_{\text{cyl}} = 3$ m. Diameter $D = 105$ m $\implies$ radius $r = 52.5$ m. $\text{CSA}_{\text{cyl}} = 2\pi r h_{\text{cyl}} = 2 \times \frac{22}{7} \times 52.5 \times 3$. $\text{CSA}_{\text{cyl}} = 2 \times 22 \times 7.5 \times 3 = 990$ m$^2$. **Conical part:** Radius $r = 52.5$ m. Slant height $l = 50$ m. $\text{CSA}_{\text{cone}} = \pi r l = \frac{22}{7} \times 52.5 \times 50$. $\text{CSA}_{\text{cone}} = 22 \times 7.5 \times 50 = 8250$ m$^2$. **Total Area and Cost:** Total area of canvas $= \text{CSA}_{\text{cyl}} + \text{CSA}_{\text{cone}} = 990 + 8250 = 9240$ m$^2$. Total cost = Total Area $\times$ Cost per m$^2$ $= 9240 \times 10 =$ Rs. 92400.

15. **Solution:** The water flowing in a minute forms a cuboid. Depth $h = 3$ m, Width $b = 40$ m. Rate of flow (length of the cuboid formed per hour) $L = 2$ km/hr $= 2000$ m/hr. Length of water falling into the sea in 1 minute $l = L \times \frac{1}{60}$ hour/min. $l = 2000 \times \frac{1}{60} = \frac{200}{6} = \frac{100}{3}$ m. Volume of water falling into the sea per minute $V = l \times b \times h$. $V = \frac{100}{3} \times 40 \times 3 = 100 \times 40 = 4000$ m$^3$.

16. **Solution:** The tank consists of a cylinder with radius $r$ and height $h$ and a hemisphere with radius $r$. Total height of the tank $H = r + h$. Therefore, $h = H - r$. Total Surface Area (TSA) of the tank: TSA = Base Area (circular) + CSA of cylinder + CSA of hemisphere. Since the base is the ground, and the cylinder is joined to the hemisphere (internal joint area is not counted), the TSA is: TSA = Area of the circular base + CSA of cylinder + CSA of hemisphere. TSA =

$\pi r^2 + 2\pi rh + 2\pi r^2$. TSA $= 3\pi r^2 + 2\pi rh$. Substitute $h = H - r$ into the equation: TSA $= 3\pi r^2 + 2\pi r(H - r)$. TSA $= 3\pi r^2 + 2\pi rH - 2\pi r^2$. TSA $= \pi r^2 + 2\pi rH = \pi r(r + 2H)$.

17. **Solution:** The actual capacity of the glass is the volume of the cylindrical glass minus the volume of the hemispherical raised portion. **Cylindrical part:** Inner diameter $D = 7$ cm $\implies$ radius $R = 3.5$ cm. Height $H = 16$ cm. Volume of the cylinder $V_{\text{cyl}} = \pi R^2 H = \frac{22}{7} \times (3.5)^2 \times 16$. $V_{\text{cyl}} = \frac{22}{7} \times 12.25 \times 16 = 22 \times 1.75 \times 16 = 616$ cm$^3$. **Hemispherical part:** Radius of the hemisphere $r = 3.5$ cm (same as the cylinder's base radius). Volume of the hemisphere $V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$. $V_{\text{hemi}} = \frac{2}{3} \times \frac{22}{7} \times 42.875 \approx 89.83$ cm$^3$. $V_{\text{hemi}} = \frac{1}{3} \times (\pi r^2) \times (2r) = \frac{1}{3} \times \left(\frac{22}{7} \times (3.5)^2\right) \times 7 = \frac{1}{3} \times 38.5 \times 7 = \frac{269.5}{3} \approx 89.833$ cm$^3$. **Actual Capacity:** Actual Capacity $V_{\text{act}} = V_{\text{cyl}} - V_{\text{hemi}} = 616 - 89.833 = 526.167$ cm$^3$.

# Section D — Case Study Solutions

**Given:** Volume of the hemispherical dome, $V = 179.67\ \text{m}^3$ Cost of insulation for inner surface = Rs. 50 per $\text{m}^2$ Cost of reflective paint for outer surface = Rs. 20 per $\text{m}^2$ $\pi = \frac{22}{7}$

**Formulae Used:**

- Volume of a hemisphere: $V = \frac{2}{3}\pi r^3$
- Curved surface area (C.S.A.) of a hemisphere: $A = 2\pi r^2$
- Area of circular base: $A_{\text{base}} = \pi r^2$

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## Q1. Finding the radius of the dome

$$V = \frac{2}{3}\pi r^3 = 179.67$$

Substitute $\pi = \frac{22}{7}$:

$$\frac{2}{3} \times \frac{22}{7} \times r^3 = 179.67$$

$$r^3 = \frac{179.67 \times 3 \times 7}{2 \times 22} = \frac{3773.07}{44} \approx 85.75$$

$$r = \sqrt[3]{85.75} \approx 4.4\ \text{m}$$

**Answer:** The radius of the dome is approximately $\boxed{4.4\ \text{m}}$.

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## Q2. Finding the inner surface area (to be insulated)

$$A = 2\pi r^2$$

$$A = 2 \times \frac{22}{7} \times (4.4)^2 = \frac{44}{7} \times 19.36 = 121.7\ \text{m}^2$$

**Answer:** The inner surface area is approximately $\boxed{122\ \text{m}^2}$.

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## Q3. Cost of reflective paint on the outer surface

$$\text{Cost} = \text{Area} \times \text{Rate}$$

$$\text{Cost} = 122 \times 20 = 2440$$

**Answer:** The cost of reflective paint is $\boxed{\text{Rs. } 2440}$.

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## Q4. Cost of insulation on the inner surface

$$\text{Cost} = 122 \times 50 = 6100$$

**Answer:** The cost of insulation is $\boxed{\text{Rs. } 6100}$.

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## Q5. Cost of mat for the circular floor

Area of the floor:

$$A_{\text{base}} = \pi r^2 = \frac{22}{7} \times (4.4)^2 = 60.8 \text{ m}^2$$

Cost of mat:

$$\text{Cost} = 60.8 \times 100 = 6080$$

**Answer:** The cost of mat to cover the floor is $\boxed{\text{Rs. } 6080}$.

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**Final Summary:**

| Q.No. | Description | Answer |
|---|---|---|
| 1 | Radius of dome | 4.4 m |
| 2 | Inner surface area | 122 $\text{m}^2$ |
| 3 | Cost of reflective paint | Rs. 2440 |
| 4 | Cost of insulation | Rs. 6100 |
| 5 | Cost of mat for floor | Rs. 6080 |