
ISC CLASS XII MATHEMATICS (TEST PAPER 1) - SET 01

Time Allowed: 3 hours

Maximum Marks: 80

Class 12 Mathematics — Answer Key

Question 1 ($10 \times 1 = 10$ marks)

1. $\boxed{3}$
2. $\boxed{625}$
3. $\boxed{\frac{3}{4}}$
4. $\boxed{k = \pm 1}$
5. $\boxed{\text{Order} = 3, \text{Degree} = 4}$
6. $\boxed{\frac{\pi}{4}}$
7. $\boxed{\text{Not one-one; Not onto (as a map } \mathbb{R} \rightarrow \mathbb{R}).}$
8. $\boxed{\text{Local maximum at } x = \frac{1}{3} \text{ with } f\left(\frac{1}{3}\right) = \frac{4}{27}; \quad \text{Local minimum at } x = 1 \text{ with } f(1) = 0.}$
9. $\boxed{\text{Yes. } P(A \cap B) = 0.24 = P(A)P(B), \text{ so } A \text{ and } B \text{ are independent.}}$
10. $\boxed{k = 0.5}$

Question 2 (3×2 Marks = 6 Marks)

1.
$$\boxed{\frac{dy}{dx} = \frac{(\cos x)^x (\ln(\cos x) - x \tan x) + x^y \cdot \frac{y}{x}}{1 - x^y \ln x}}$$
2. $\boxed{\text{There exists } c = 2.5 \in (1, 4) \text{ such that } f'(c) = \frac{f(4) - f(1)}{4 - 1} = 1.}$
3. $\boxed{(P(A), P(B)) = \left(\frac{1}{2}, \frac{2}{3}\right) \text{ or } \left(\frac{1}{3}, \frac{1}{2}\right).}$

Question 3 ($4 \times 4 = 16$ marks)

1. $\frac{1}{a}$
2. Tangent: $y + 4 = \frac{3}{4}(x - 3)$; Normal: $y + 4 = -\frac{4}{3}(x - 3)$.
3. 0
4. $-(a - b)(b - c)(c - a)(a + b + c)$

Question 4 ($3 \times 6 = 18$ marks)

1. Semi-vertical angle $\alpha = \tan^{-1}(\sqrt{2})$.
2. $\ln |x - 1| - 5 \ln |x - 2| + 4 \ln |x - 3| + C$.
3. $y(x) = x^2 + C \csc x$, where C is an arbitrary constant (and $\sin x \neq 0$).

Question 5 (15 Marks)

1. (a) The function $f(x) = \sin x$ on $\mathbb{R}$ is not invertible. Restrict the domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (or any interval of length π).
Then $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$ is bijective and $f^{-1} = \arcsin$.
2. (b) $(x, y, z) = (-13, 6, -2)$.
3. (c) $\frac{11}{243}$

SECTION B

Question 6 (5 Marks)

1. (a) $\text{proj}_{\mathbf{b}} \mathbf{a} = \frac{5}{3} \mathbf{b} = \left(\frac{5}{3}, \frac{10}{3}, \frac{5}{3} \right).$
2. (b) A vector of magnitude 9 perpendicular to both is $\pm (-3, 6, 6).$

Question 7 (10 Marks)

1. $51x + 15y - 50z + 173 = 0.$
2. $A = \frac{16\pi + 4\sqrt{3}}{3} = \frac{4}{3}(4\pi + \sqrt{3}).$

SECTION C (15 Marks)

Question 8 (5 Marks)

1. $P(x) = 5x - 3x^2 - 10, \quad P_{\max} \text{ occurs at } x = \frac{5}{6}, \quad P_{\max} = -\frac{95}{12} (\approx -7.9167).$

Question 9 (10 Marks)

- (a) Maximum value $Z_{\max} = 18$ at $(x, y) = (0, 9).$
- (b) $\hat{y} = a + bx, \quad b = \frac{179}{178} \approx 1.005618, \quad a = \frac{2476}{445} \approx 5.564045.$ Estimated physics marks for $x = 48 :$