ISC CLASS XII MATHEMATICS (TEST PAPER 6) - SET 06

Time Allowed: 3 hours **Maximum Marks: 80**

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. **Question:** Let $*$ be an operation on $\mathbb{Z}$ defined by $a * b = |a - b|$. Check if $*$ is commutative.

   Answer: Yes, $*$ is commutative.

   Solution: An operation $*$ is commutative if $a * b = b * a$ for all $a, b \in \mathbb{Z}$. Given $a * b = |a - b|$ and $b * a = |b - a|$. Since $|a - b| = |b - a|$, it follows that $a * b = b * a$. Thus, $*$ is commutative.

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2. **Question:** Find the principal value of $\sec^{-1}(-2)$.

   Answer: $\frac{2\pi}{3}$

   Solution: The range of $\sec^{-1}(x)$ is $[0, \pi] \setminus \{\frac{\pi}{2}\}$. $\sec\theta = -2$ implies $\cos\theta = -\frac{1}{2}$. The principal value of θ in $[0, \pi]$ satisfying $\cos\theta = -\frac{1}{2}$ is $\frac{2\pi}{3}$.

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3. **Question:** If $f(x) = |x| - 5$ and $g(x) = x^2 + 1$, find $f \circ g(1)$.

   Answer: -3

   Solution: First, compute $g(1) = (1)^2 + 1 = 2$. Now,
   $f \circ g(1) = f(g(1)) = f(2) = |2| - 5 = 2 - 5 = -3$.

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4. **Question:** State the range of the function $f(x) = \frac{\pi}{2} - \cos^{-1} x$.

   Answer: $[0, \pi]$

   Solution: The range of $\cos^{-1} x$ is $[0, \pi]$. Thus, the range of $f(x) = \frac{\pi}{2} - \cos^{-1} x$ is:
   $\frac{\pi}{2} - \pi \leq f(x) \leq \frac{\pi}{2} - 0 - \frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$ However, since $\cos^{-1} x$ ranges from 0 to π, $f(x)$ ranges from $\frac{\pi}{2} - \pi$ to $\frac{\pi}{2} - 0$, i.e., $[-\frac{\pi}{2}, \frac{\pi}{2}]$. But the question likely expects the range of $\frac{\pi}{2} - \cos^{-1} x$ as $[0, \pi]$ if interpreted as $\sin^{-1} x$. *Correction:* The range of $f(x) = \frac{\pi}{2} - \cos^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. *Alternative Question:* State the range of $f(x) = \sin^{-1} x$.

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5. **Question:** If $x^2 + y^2 = 5$, find $\frac{dy}{dx}$ at $(1, 2)$.

   Answer: -1

   Solution: Differentiate implicitly: $2x + 2y\frac{dy}{dx} = 0$. Solve for $\frac{dy}{dx}$: $\frac{dy}{dx} = -\frac{x}{y}$. At $(1, 2)$, $\frac{dy}{dx} = -\frac{1}{2}$. *Correction:* The point $(1, 2)$ does not satisfy $x^2 + y^2 = 5$. *Alternative Question:* If $x^2 + y^2 = 5$, find $\frac{dy}{dx}$ at $(1, 2)$ is invalid. Use $(1, 2)$ for $x^2 + y^2 = 5$ is incorrect. Use $(1, 2)$ for $x^2 + y^2 = 5$ is not possible. Use $(1, 2)$ for $x^2 + y^2 = 5$ is not valid. *Alternative Question:* If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$ at $(3, 4)$. *Answer:* $-\frac{3}{4}$

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6. **Question:** Evaluate: $\int e^x(1 + x)\,dx$.

   Answer: $e^x(x) + C$

   Solution: Let $u = x$ and $dv = e^x dx$. Then $du = dx$ and $v = e^x$. Using integration by parts:
   $\int u\,dv = uv - \int v\,du$. Thus, $\int e^x(1 + x)\,dx = \int e^x dx + \int xe^x dx = e^x + (xe^x - e^x) + C = xe^x + C$.

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7. **Question:** What is the degree of the homogeneous differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$?

   Answer: 1

   Solution: A differential equation is homogeneous if it can be written as $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$. Here, $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$, which is homogeneous of degree 1.

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8. **Question:** If $P(A) = 0.4$ and $P(A \cap B) = 0.1$, find $P(A \cap B')$.

   Answer: 0.3

   Solution: $P(A \cap B') = P(A) - P(A \cap B) = 0.4 - 0.1 = 0.3$.

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9. **Question:** If the mean of a probability distribution is 10, what is $E(2X + 3)$?

   Answer: 23

   Solution: $E(2X + 3) = 2E(X) + 3 = 2(10) + 3 = 23$.

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10. **Question:** If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, find a non-zero row matrix X such that $XA = 0$.

    Answer: $X = \begin{pmatrix} 2 & -1 \end{pmatrix}$

    Solution: Let $X = \begin{pmatrix} x & y \end{pmatrix}$. Then $XA = \begin{pmatrix} 2x + y & x + 4y \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$. This gives the system: $2x + y = 0$ and $x + 4y = 0$. Solving, we get $x = 2k$ and $y = -k$ for any $k \neq 0$. Thus, $X = \begin{pmatrix} 2 & -1 \end{pmatrix}$ is a solution.

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. **Question:** If $x = \sin^3 t$ and $y = \cos^3 t$, find $\frac{dy}{dx}$.

   Answer: $\frac{dy}{dx} = -\tan t$

   Solution: Given $x = \sin^3 t$ and $y = \cos^3 t$. Differentiate both with respect to t: $\frac{dx}{dt} = 3\sin^2 t \cos t$ and $\frac{dy}{dt} = -3\cos^2 t \sin t$. Thus, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\cos^2 t \sin t}{3\sin^2 t \cos t} = -\frac{\cos t}{\sin t} = -\cot t$. *Correction:* $\frac{dy}{dx} = -\tan t$ is incorrect. The correct answer is $\frac{dy}{dx} = -\cot t$.

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2. **Question:** A balloon is spherical in shape. Gas is leaking out of the balloon at the rate of 10 cm^3/min. How fast is the radius of the balloon decreasing when the radius is 15 cm?

   Answer: $\frac{1}{90\pi}$ cm/min

   Solution: The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Differentiate with respect to t: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Given $\frac{dV}{dt} = -10$ and $r = 15$: $-10 = 4\pi(15)^2 \frac{dr}{dt}$. Thus, $\frac{dr}{dt} = \frac{-10}{4\pi(225)} = \frac{-1}{90\pi}$ cm/min. The radius is decreasing at $\frac{1}{90\pi}$ cm/min.

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3. **Question:** A bag contains 4 red and 5 black balls. Two balls are drawn at random without replacement. Find the probability that both balls are red.

   Answer: $\frac{2}{9}$

   Solution: Total balls $= 4 + 5 = 9$. Probability of drawing two red balls: $P(\text{first red}) = \frac{4}{9}$ and $P(\text{second red}) = \frac{3}{8}$. Thus, $P(\text{both red}) = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$. *Correction:* The correct probability is $\frac{1}{6}$.

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Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. **Question:** Find a matrix X such that $2A + B + X = 0$, where $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$.

   Answer: $X = \begin{pmatrix} -5 & 2 \\ -7 & -13 \end{pmatrix}$

   Solution: $2A + B + X = 0 \implies X = -2A - B$. Compute $2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$. Thus,
   $X = -2A - B = \begin{pmatrix} -2 & -4 \\ -6 & -8 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -7 & -13 \end{pmatrix}$.

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2. **Question:** Evaluate: $\int \frac{dx}{x^2-6x+13}$.

   Answer: $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$

   Solution: Complete the square: $x^2 - 6x + 13 = (x-3)^2 + 4$. Let $u = x - 3$, then
   $\int \frac{dx}{u^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$.

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3. **Question:** Show that $f(x) = \log(\cos x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

   Solution: Compute the derivative: $f'(x) = \frac{-\sin x}{\cos x} = -\tan x$. Since $\tan x > 0$ for $x \in \left(0, \frac{\pi}{2}\right)$, $f'(x) = -\tan x < 0$. Thus, $f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

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4. **Question:** Solve the differential equation: $\frac{dy}{dx} - y = \cos x - \sin x$.

   Answer: $y = \sin x + Ce^x$

   Solution: This is a linear differential equation. The integrating factor is $e^{\int -1\,dx} = e^{-x}$. Multiply through: $e^{-x}\frac{dy}{dx} - e^{-x}y = e^{-x}(\cos x - \sin x)$. The left side is $\frac{d}{dx}(e^{-x}y)$, so:
   $e^{-x}y = \int e^{-x}(\cos x - \sin x)\,dx = e^{-x}\sin x + C$. Thus, $y = \sin x + Ce^x$.

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Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. **Question:** An open tank with a square base and vertical sides is to be constructed from a metal sheet of given area A. Show that the cost of material will be least if the depth is half of the width.

   Solution: Let the side of the square base be x and the depth be h. The surface area
   $S = x^2 + 4xh = A$. Express h in terms of x: $h = \frac{A-x^2}{4x}$. The volume
   $V = x^2 h = x^2\left(\frac{A-x^2}{4x}\right) = \frac{Ax-x^3}{4}$. To minimize the cost (i.e., minimize S), maximize V.
   Differentiate V with respect to x and set to zero: $\frac{dV}{dx} = \frac{A-3x^2}{4} = 0 \implies x = \sqrt{\frac{A}{3}}$. Substitute back to find $h = \frac{A-\frac{A}{3}}{4\sqrt{\frac{A}{3}}} = \frac{\frac{2A}{3}}{4\sqrt{\frac{A}{3}}} = \frac{\sqrt{A}}{3\sqrt{3}} = \frac{x}{2}$. Thus, the depth is half the width.

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2. **Question:** Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$.

   Answer: $\frac{\pi^2}{4}$

Solution: Let $I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$. Use the substitution $x = \pi - t$:
$I = \int_0^\pi \frac{(\pi-t)\sin t}{1+\cos^2 t}dt = \pi \int_0^\pi \frac{\sin t}{1+\cos^2 t}dt - I$. Thus, $2I = \pi \int_0^\pi \frac{\sin t}{1+\cos^2 t}dt$. Let $u = \cos t$, then $du = -\sin t\, dt$: $2I = \pi \int_{-1}^1 \frac{-du}{1+u^2} = \pi \left[\tan^{-1} u\right]_{-1}^1 = \pi \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi^2}{2}$. Thus, $I = \frac{\pi^2}{4}$.

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3. **Question:** Solve the system of linear equations using the matrix inverse method:

$$x + 2y + z = 4$$
$$-x + y + z = 0$$
$$x - 3y + z = 2$$

Answer: $x = 1$, $y = 1$, $z = 1$

Solution: The system is $AX = B$, where $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$. The inverse of A is $A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ -2 & 5 & 3 \end{pmatrix}$. Thus,
$X = A^{-1}B = \frac{1}{10}\begin{pmatrix} 4(4) - 5(0) + 1(2) \\ 2(4) + 0(0) - 2(2) \\ -2(4) + 5(0) + 3(2) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

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Question 5 (15 Marks)

Answer the following questions.

1. **(a)** If $f : \mathbb{R} \to [4, \infty)$ is defined by $f(x) = x^2 + 4$, show that f is not invertible. Restrict the domain to $D = [0, \infty)$ and find f^{-1}.

   Solution: f is not invertible because it is not one-to-one (e.g., $f(1) = f(-1) = 5$). Restrict f to $D = [0, \infty)$. Then f is one-to-one and onto $[4, \infty)$. To find f^{-1}, solve $y = x^2 + 4$ for x: $x = \sqrt{y-4}$. Thus, $f^{-1}(y) = \sqrt{y-4}$.

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2. **(b)** Find the mean and variance of the number of doubles when a pair of dice is thrown 3 times.

   Answer: Mean = 0.5, Variance = 0.4167

   Solution: The probability of a double in one throw is $\frac{6}{36} = \frac{1}{6}$. Let X be the number of doubles in 3 throws. Then $X \sim \text{Binomial}(n = 3, p = \frac{1}{6})$. Mean $= np = 3 \times \frac{1}{6} = 0.5$. Variance $= np(1-p) = 3 \times \frac{1}{6} \times \frac{5}{6} = \frac{15}{36} = 0.4167$.

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3. **(c)** In a class, 40% students study Mathematics and 30% study Biology. 10% study both. Find the probability that a student studies Mathematics given he/she studies Biology.

   Answer: $\frac{1}{3}$

   Solution: Let M be the event of studying Mathematics and B be the event of studying Biology. Given $P(M) = 0.4$, $P(B) = 0.3$, and $P(M \cap B) = 0.1$. $P(M|B) = \frac{P(M\cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$.

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. **Question:** Find the area of the parallelogram whose diagonals are $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$.

   Answer: $\frac{\sqrt{651}}{2}$

   Solution: The area of the parallelogram formed by the diagonals $\vec{d_1}$ and $\vec{d_2}$ is given by:

   $$\text{Area} = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$$

   Compute the cross product:

   $$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \hat{i}(1 \cdot 4 - (-2) \cdot (-3)) - \hat{j}(3 \cdot 4 - (-2) \cdot 1) + \hat{k}(3 \cdot (-3) - 1 \cdot 1)$$

   $$= \hat{i}(4 - 6) - \hat{j}(12 + 2) + \hat{k}(-9 - 1) = -2\hat{i} - 14\hat{j} - 10\hat{k}$$

   The magnitude of the cross product is:

   $$|\vec{d_1} \times \vec{d_2}| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \sqrt{4 + 196 + 100} = \sqrt{300} = 10\sqrt{3}$$

   Thus, the area of the parallelogram is:

   $$\frac{1}{2} \times 10\sqrt{3} = 5\sqrt{3}$$

   Correction: The magnitude is $\sqrt{4 + 196 + 100} = \sqrt{300} = 10\sqrt{3}$. The area is $\frac{1}{2} \times 10\sqrt{3} = 5\sqrt{3}$. *Note:* The original answer was incorrect; the correct area is $5\sqrt{3}$.

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2. **Question:** Find the scalar product of $(\vec{a} + 3\vec{b})$ and $(2\vec{a} - \vec{b})$, if $|\vec{a}| = 2$, $|\vec{b}| = 3$, and $\vec{a} \cdot \vec{b} = 1$.

   Answer: 1

   Solution: The scalar product is:

   $$(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b}) = 2(\vec{a} \cdot \vec{a}) - (\vec{a} \cdot \vec{b}) + 6(\vec{b} \cdot \vec{a}) - 3(\vec{b} \cdot \vec{b})$$

   Substitute the given values:

   $$= 2(2^2) - 1 + 6(1) - 3(3^2) = 2(4) - 1 + 6 - 27 = 8 - 1 + 6 - 27 = -14$$

   Correction: The correct scalar product is -14. *Note:* The original answer was incorrect; the correct scalar product is -14.

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Question 7 (10 Marks)

Answer the following questions.

1. **Question:** Find the distance of the origin from the plane $3x - 4y + 12z = 52$. Find the vector equation of the plane.

   Answer: Distance = 4; Vector equation: $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 52$

Solution: The distance D of the origin from the plane $3x - 4y + 12z = 52$ is:

$$D = \frac{|3(0) - 4(0) + 12(0) - 52|}{\sqrt{3^2 + (-4)^2 + 12^2}} = \frac{52}{\sqrt{9 + 16 + 144}} = \frac{52}{\sqrt{169}} = \frac{52}{13} = 4$$

The vector equation of the plane is:

$$\vec{r} \cdot (3\hat{\imath} - 4\hat{\jmath} + 12\hat{k}) = 52$$

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2. **Question:** Using integration, find the area bounded by the parabola $y^2 = 4x$ and the latus rectum.

   Answer: $\frac{8}{3}$

   Solution: The parabola $y^2 = 4x$ has its latus rectum at $x = 1$. The points of intersection of the parabola and the latus rectum are $(1, 2)$ and $(1, -2)$. The area is:

$$\text{Area} = \int_{-2}^{2} \sqrt{4 - y^2}\, dy$$

   But since $x = \frac{y^2}{4}$, the area is:

$$\text{Area} = \int_{-2}^{2} \frac{y^2}{4}\, dy = 2\int_{0}^{2} \frac{y^2}{4}\, dy = 2\left[\frac{y^3}{12}\right]_0^2 = 2\left(\frac{8}{12}\right) = \frac{16}{12} = \frac{4}{3}$$

   Correction: The correct area is $\frac{8}{3}$. *Note:* The correct limits and integrand should be:

$$\text{Area} = \int_{0}^{1} 2\sqrt{4x}\, dx = 2\int_{0}^{1} 2\sqrt{x}\, dx = 4\left[\frac{x^{3/2}}{3/2}\right]_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$$

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. **Question:** The demand function for a certain commodity is $p = 100 - 4x$ and the cost function is $C(x) = 50x + 200$. Find the profit function $P(x)$, the marginal profit, and the output level x at which the marginal revenue is zero.

   Answer: Profit function: $P(x) = 50x - 4x^2 - 200$, Marginal profit: $P'(x) = 50 - 8x$, Output level at which marginal revenue is zero: $x = 12.5$

   Solution: The revenue function $R(x)$ is:

$$R(x) = p \cdot x = (100 - 4x) \cdot x = 100x - 4x^2$$

   The profit function $P(x)$ is:

$$P(x) = R(x) - C(x) = (100x - 4x^2) - (50x + 200) = 50x - 4x^2 - 200$$

   The marginal profit is the derivative of $P(x)$:

$$P'(x) = 50 - 8x$$

The marginal revenue is the derivative of $R(x)$:

$$R'(x) = 100 - 8x$$

Set the marginal revenue to zero to find the output level:

$$100 - 8x = 0 \implies x = \frac{100}{8} = 12.5$$

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Question 9 (10 Marks)

Answer the following questions.

1. **Question:** Solve the following Linear Programming Problem graphically: Minimize $Z = 2x + 3y$ Subject to the constraints:

$$\begin{aligned} x + 2y &\leq 10 \\ 2x + y &\leq 8 \\ x, y &\geq 0 \end{aligned}$$

Answer: The minimum value of Z is 12 at the point $(4, 2)$.

Solution: Plot the constraints:

- $x + 2y \leq 10$ intersects the axes at $(10, 0)$ and $(0, 5)$.
- $2x + y \leq 8$ intersects the axes at $(4, 0)$ and $(0, 8)$.

The feasible region is a polygon with vertices at $(0, 0)$, $(4, 0)$, $(4, 3)$, $(0, 5)$, and $(0, 0)$. Evaluate $Z = 2x + 3y$ at each vertex:

- At $(0, 0)$: $Z = 0$
- At $(4, 0)$: $Z = 8$
- At $(4, 3)$: $Z = 17$
- At $(0, 5)$: $Z = 15$
- At $(2, 4)$: $Z = 16$
- At $(4, 2)$: $Z = 12$

The minimum value of Z is 12 at the point $(4, 2)$.

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2. **Question:** For a bivariate distribution, the two regression coefficients are $b_{yx} = -0.5$ and $b_{xy} = -0.8$. If the variance of y is 16, find the coefficient of correlation (r) and the standard deviation of x.

Answer: Coefficient of correlation $(r) = -0.6325$, Standard deviation of $x = 5$

Solution: The relationship between the regression coefficients and the correlation coefficient is:

$$b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y} \quad \text{and} \quad b_{xy} = r \cdot \frac{\sigma_y}{\sigma_x}$$

Given $b_{yx} = -0.5$ and $b_{xy} = -0.8$, we have:

$$b_{yx} \cdot b_{xy} = r^2 \implies (-0.5) \cdot (-0.8) = r^2 \implies r^2 = 0.4 \implies r = \pm\sqrt{0.4} = \pm 0.6325$$

Since both regression coefficients are negative, r is negative:

$$r = -0.6325$$

Given $\sigma_y^2 = 16 \implies \sigma_y = 4$. Using $b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y}$:

$$-0.5 = -0.6325 \cdot \frac{\sigma_x}{4} \implies \sigma_x = \frac{0.5 \cdot 4}{0.6325} \approx 3.162$$

Correction:

$$b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y} \implies -0.5 = -0.6325 \cdot \frac{\sigma_x}{4} \implies \sigma_x = \frac{0.5 \cdot 4}{0.6325} \approx 3.162$$

Note: The correct standard deviation of x is approximately 3.162. However, if we use the exact value of r:

$$r = -\sqrt{0.4} = -\frac{2}{\sqrt{5}}$$

$$b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y} \implies -0.5 = -\frac{2}{\sqrt{5}} \cdot \frac{\sigma_x}{4} \implies \sigma_x = \frac{0.5 \cdot 4 \cdot \sqrt{5}}{2} = \sqrt{5} \approx 2.236$$

Correction: The correct standard deviation of x is $\sqrt{5}$.