ISC CLASS XII MATHEMATICS (TEST PAPER 19) - SET 19

Time Allowed: 3 hours **Maximum Marks: 80**

SECTION A

Question 1

1. $*$ is **not associative**.
2. $\sin\left(\cos^{-1}\frac{3}{5}\right) = \frac{4}{5}$.
3. The domain is $\{0\}$ (only $x = 0$).
4. $f^{-1}(a) = 1,\ f^{-1}(b) = 2,\ f^{-1}(c) = 3$.
5. $\dfrac{dy}{dx} = \dfrac{\cot x}{2\sqrt{\ln(\sin x)}}$.
6. $\displaystyle\int_0^{\pi} \cos x\, dx = 0$.
7. Order $= 2$. Degree: **not defined**.
8. The value of k is 3.
9. A and B are **independent**.
10. $(A^T)^T = A$.

Question 2

1. $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2a\,t^3} = -\dfrac{\sqrt{a}}{2\,x^{3/2}}$.
2. $\dfrac{dV}{dt} = \dfrac{8}{15}\,\pi$ cm^3/s.
3. The probability is $\dfrac{3}{5}$.

Question 3

1. Tangent: $y = 4x - 4$. Normal: $y = -\dfrac{1}{4}x + \dfrac{9}{2}$.
2. The particular solution is $y(x) = x\,\sinh\left(2\ln x\right) = \dfrac{x^3 - x^{-1}}{2}$.
3. $\displaystyle\int \frac{x}{(x+1)(x^2+1)}\,dx = -\frac{1}{2}\ln|x+1| + \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{2}\tan^{-1}x + C$.
4. $X = \begin{pmatrix} 1 & 2 \\ -\frac{1}{3} & 0 \end{pmatrix}$.

Question 4

1. The determinant equals $(1 + pxyz)\,(x - y)(y - z)(z - x)$.
2. The nearest point is approximately $(1.5494,\ 3.5184)$.
3. $\displaystyle\int \frac{dx}{3x^2 + 13x - 10} = \frac{3}{17} \ln \left| \frac{x - \frac{2}{3}}{x + 5} \right| + C.$

Question 5

(a) R is an equivalence relation on $\mathbb{Z}$.

(b) $x(y) = \dfrac{y^3}{4} + \dfrac{C}{y}.$

(c) The probability is $\dfrac{1}{4} = 0.25 = 25\%$.

SECTION B

Question 6

1. $\text{proj}_{\vec{b}}\,\vec{a} = \dfrac{10}{19}(7\hat{i} - \hat{j} + 8\hat{k}) = \dfrac{70}{19}\,\hat{i} - \dfrac{10}{19}\,\hat{j} + \dfrac{80}{19}\,\hat{k}.$
2. The area is $\sqrt{13}$.

Question 7

1. The distance between the parallel planes is $\dfrac{1}{6}$.
2. The area is 9.

SECTION C

Question 8

1. The optimal box is a cube with side $\sqrt{\dfrac{200}{3}} \approx 8.16497$ cm and height $\sqrt{\dfrac{200}{3}} \approx 8.16497$ cm.

Question 9

1. The minimum value of Z is 0 at $(0, 0)$.
2. $\sigma_x = 5.6$.