PRACTICE QUESTION PAPER - II

CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each.
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. If the function $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2 + 1$, then $f^{-1}(17)$ is equal to:
   (a) 4
   (b) $\{-4, 4\}$
   (c) 3
   (d) $\{3, -3\}$
2. Let $A = \{1, 2, 3\}$. The total number of non-empty subsets of A is:
   (a) 6
   (b) 7
   (c) 8
   (d) 9
3. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is:
   (a) π
   (b) 2π
   (c) $\frac{3\pi}{4}$
   (d) $\frac{5\pi}{4}$
4. If $f(x) = \frac{x}{x-1}, x \neq 1$, then $f(f(x))$ is equal to:
   (a) x
   (b) $\frac{1}{x}$
   (c) x^2
   (d) $\frac{x}{(x-1)^2}$

5. The range of $\sec^{-1} x$ is:

   (a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

   (b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

   (c) $(-\infty, \infty)$

   (d) $[-\pi, \pi]$

6. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, then A^4 is equal to:

   (a) $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

   (b) $\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

   (c) $\begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$

   (d) $\begin{bmatrix} 2^4 & 2^4 \\ 2^4 & 2^4 \end{bmatrix}$

7. If A is a skew-symmetric matrix, then A^T is equal to:

   (a) A

   (b) $-A$

   (c) I

   (d) 0

8. If A is a matrix of order 3×4, then each row of A has:

   (a) 3 elements

   (b) 4 elements

   (c) 7 elements

   (d) 12 elements

9. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is:

   (a) 3

   (b) 4

   (c) ± 6

   (d) ± 3

10. If A is a non-singular matrix, then $adj(adj(A))$ is equal to:

    (a) $|A|A$

    (b) $|A|^2 A$

    (c) $|A|^{n-2} A$ (where n is order)

    (d) $|A|^3 A$

11. The function $f(x) = \sin x + \cos x$ is increasing in the interval:

    (a) $\left(0, \frac{\pi}{2}\right)$

    (b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

    (c) $\left(0, \frac{\pi}{4}\right)$

(d) $\left(\frac{\pi}{2}, \pi\right)$

12. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:
    (a) $e^y = e^x + \frac{x^3}{3} + C$
    (b) $e^y = e^x + \frac{x^3}{3}$
    (c) $e^{-y} = e^x + \frac{x^3}{3} + C$
    (d) $e^y = e^x + x^3 + C$
13. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to:
    (a) $25y$
    (b) $5y$
    (c) $-25y$
    (d) $10y$
14. The value of $\int_1^e \log x \, dx$ is:
    (a) $e - 1$
    (b) 1
    (c) 0
    (d) $\frac{1}{e}$
15. The area bounded by $y = |x|$ and $y = 1$ is:
    (a) 1 sq. unit
    (b) 2 sq. units
    (c) $\frac{1}{2}$ sq. unit
    (d) 4 sq. units
16. If $\mathbf{r} = \begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$, then $\frac{\partial \mathbf{r}}{\partial z}$ is:
    (a) $4x + y - 2z$
    (b) -3
    (c) -3
    (d) $5 - 8$
17. The angle between the vector $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and the plane $\mathbf{r} \cdot (\hat{i} + \hat{k}) = 1$ is:
    (a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
    (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
    (c) $\sin^{-1}\left(\frac{1}{3}\right)$
    (d) 0
18. The point of intersection of the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and the xy-plane is:
    (a) $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$
    (b) $(0, 1, -2)$
    (c) $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$
    (d) $\left(-\frac{2}{3}, \frac{1}{3}, 0\right)$

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The maximum value of $\frac{\log x}{x}$ is $\frac{1}{e}$. **Reason (R):** $\frac{d}{dx}\left(\frac{\log x}{x}\right) = \frac{1-\log x}{x^2}$.

20. **Assertion (A):** The binary operation $*$ defined on $\mathbb{Z}$ by $a * b = ab + 1$ is commutative. **Reason (R):** An operation $*$ is commutative if $a * b = b * a$ for all $a, b \in \mathbb{Z}$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + x^{\sin x}$.

22. Given $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Show that $A^2 - 6A + 11I = 0$, where I is the identity matrix.

23. Find the magnitude of $\vec{a} \times \vec{b}$, if $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

OR

Find the area of a parallelogram whose adjacent sides are represented by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

24. Find the interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly decreasing.

25. A bag contains 5 red and 3 black balls. If three balls are drawn at random without replacement, find the probability that exactly two balls are red.

OR

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, find $P(A|B)$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

27. Evaluate $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx$.

28. Find the coordinates of the point where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{-2}$ intersects the xz-plane.

OR

Find the angle between the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

29. Find the general solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$.

OR

Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.

30. Express the matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

31. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$.

32. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ subject to the constraints $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

OR

A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, Automatic and Hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machine to manufacture a packet of screw A, while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screw B. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screw A at a profit of $Rs.7$ and screw B at a profit of $Rs.10$. Assuming that he can sell all the screws he manufactures, how many packets of each type should be made in a day to maximize his profit? (Formulate as LPP).

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

34. Using the method of integration, find the area of the triangular region whose vertices are $(1, 0)$, $(2, 2)$, and $(3, 1)$.

35. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$.

OR

Find the derivative of $\sin x$ with respect to x^x.

36. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

OR

Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

37. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

39. **Case Study 1: Production Cost and Marginal Cost**

A company manufactures electronic devices. The cost function for manufacturing x units of a product is given by $C(x) = 500 + 10x + 0.005x^2$. The Marginal Cost (MC) is the instantaneous rate of change of total cost $C(x)$ with respect to the number of units produced x.

Based on the given information, answer the following questions:

(a) Find the Marginal Cost function $MC(x)$. (1 Mark)

(b) Calculate the Marginal Cost when 100 units are produced. (1 Mark)

(c) Determine the number of units x for which the Average Cost is minimum. (2 Marks)

OR

(d) Find the number of units x at which the Marginal Cost is equal to the Average Cost. (2 Marks)

40. **Case Study 2: Bridge Construction and Skew Lines**

An engineer is planning the design for a cable-stayed bridge. Two main cable segments are modeled as two lines L_1 and L_2 in space. Line L_1 passes through $P(3, 1, 5)$ and is parallel to the vector $\vec{b}_1 = \hat{i} + \hat{j} - \hat{k}$. Line L_2 passes through $Q(4, -2, 3)$ and is parallel to the vector $\vec{b}_2 = 2\hat{i} + \hat{j} + 3\hat{k}$.

Based on the given information, answer the following questions:

(a) Write the vector equation of line L_1. (1 Mark)

(b) Show that the lines L_1 and L_2 are skew lines. (1 Mark)

(c) Find the dot product of the direction vectors $\vec{b}_1$ and $\vec{b}_2$. (2 Marks)

OR

(d) Find the vector perpendicular to both lines L_1 and L_2. (2 Marks)

41. **Case Study 3: Bernoulli Trials and Binomial Distribution**

In a game of archery, the probability of a player hitting the target is $\frac{1}{4}$. The player is allowed to shoot 5 arrows independently.

Based on the given information, answer the following questions:

(a) What is the probability of the player hitting the target exactly 3 times? (1 Mark)

(b) Find the probability of the player hitting the target at least once. (2 Marks)

OR

(c) If the player shoots n arrows, and the probability of hitting the target at least once is greater than $\frac{1}{2}$, find the minimum value of n. (2 Marks)