PRACTICE QUESTION PAPER - XVIII
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let R be a relation on the set $\mathbb{N}$ of natural numbers defined by $x + 2y = 8$. The range of R is:
   (a) $\{1, 2, 3, 4, 6\}$
   (b) $\{1, 2, 3\}$
   (c) $\{2, 4, 6\}$
   (d) $\{1, 3\}$
2. If $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = (3 - x^3)^{1/3}$, then $f \circ f(x)$ is:
   (a) x^3
   (b) x
   (c) $3 - x^3$
   (d) $(3 - x^3)^{1/3}$
3. The value of $\sin(\tan^{-1} x)$, $|x| < 1$, is:
   (a) $\frac{x}{\sqrt{1-x^2}}$
   (b) $\frac{1}{\sqrt{1+x^2}}$
   (c) $\frac{x}{\sqrt{1+x^2}}$
   (d) $\sqrt{1 - x^2}$
4. The principal value of $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is:
   (a) $\pi/6$
   (b) $\pi/4$
   (c) $\pi/3$

(d) $\pi/2$

5. If A is a skew-symmetric matrix, then the diagonal elements of A are:
   (a) All non-zero
   (b) All 1
   (c) All 0
   (d) None of these

6. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then the matrix $A - A^T$ is:
   (a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$
   (d) $\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$

7. If A is a square matrix of order 3 such that $|adj A| = 64$, then $|A|$ is:
   (a) ± 8
   (b) 8
   (c) 64
   (d) ± 4

8. If the points $(x, -2), (5, 2), (8, 8)$ are collinear, then x is equal to:
   (a) 1
   (b) -1
   (c) 2
   (d) 0

9. The derivative of $\sin^2(\sqrt{x})$ with respect to x is:
   (a) $\frac{\sin(2\sqrt{x})}{2\sqrt{x}}$
   (b) $\cos^2(\sqrt{x})$
   (c) $\frac{2\sin(\sqrt{x})\cos(\sqrt{x})}{x}$
   (d) $\frac{\sin(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

10. The equation of the tangent to the curve $y = x^2 + 3x + 4$ at $x = 1$ is:
    (a) $5x - y + 7 = 0$
    (b) $5x - y + 1 = 0$
    (c) $5x - y - 1 = 0$
    (d) $x + 5y - 7 = 0$

11. $\int \frac{1}{\sin^2 x \cos^2 x}\, dx$ is equal to:
    (a) $\tan x + \cot x + C$
    (b) $\tan x - \cot x + C$
    (c) $-\tan x + \cot x + C$

(d) $\sec x - \csc x + C$

12. The value of $\int_0^1 xe^{x^2}\,dx$ is:
    (a) $e - 1$
    (b) $\frac{e-1}{2}$
    (c) e
    (d) 1
13. The degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + y = x\frac{dy}{dx}$ is:
    (a) 1
    (b) 2
    (c) 3
    (d) Not defined
14. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, then $\vec{a} \cdot \vec{b}$ is:
    (a) 1
    (b) 2
    (c) -1
    (d) 0
15. The area of a parallelogram whose adjacent sides are $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = \hat{\imath} + 4\hat{\jmath}$ is:
    (a) 5 sq units
    (b) 6 sq units
    (c) 3 sq units
    (d) 11 sq units
16. The distance of the plane $3x - 4y + 12z = 52$ from the origin is:
    (a) 4 units
    (b) 52 units
    (c) 13 units
    (d) 12 units
17. The direction cosines of a line equally inclined to the coordinate axes are:
    (a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
    (b) $1, 1, 1$
    (c) $0, 0, 0$
    (d) $1, 0, 0$
18. The corner points of the feasible region determined by $x + y \leq 2, x \geq 0, y \geq 0$ are:
    (a) $(0, 0), (2, 0), (0, 2), (1, 1)$
    (b) $(0, 0), (2, 0), (0, 2)$
    (c) $(0, 0), (2, 0)$
    (d) $(0, 0), (0, 2)$

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If $P(A) = 0.4$ and $P(B) = 0.5$ and A and B are mutually exclusive, then $P(A \cup B) = 0.9$. **Reason (R):** If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

20. **Assertion (A):** The derivative of $\log(\sec x + \tan x)$ is $\sec x$. **Reason (R):** $\frac{d}{dx}(\log f(x)) = \frac{1}{f(x)} \cdot f'(x)$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \log(x + \sqrt{x^2 + a^2})$.

22. Find the area of the triangle with vertices $A(1, 1, 1), B(1, 2, 3)$ and $C(2, 3, 1)$.

OR

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle.

23. Evaluate $\int \frac{dx}{9x^2+6x+5}$.

OR

Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

24. Construct a 3×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$.

25. If $P(A') = 0.3, P(B) = 0.4$ and $P(A \cap B') = 0.5$. Find $P(A \cup B)$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Check whether the relation R defined in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

27. Evaluate $\int \frac{2x-3}{(x^2-1)(2x+3)}\, dx$.

OR

Evaluate $\int \frac{\cos x - \sin x}{1 + \sin 2x}\, dx$.

28. Find the general solution of the differential equation $e^{x/y}(x\frac{dy}{dx} - y) = x$.

OR

Find the interval in which the function $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 + 5$ is strictly decreasing.

29. Find the equation of the plane passing through the point $(-1, 3, 2)$ and parallel to the plane $x + 2y + 3z = 5$. Also find the distance between the two planes.

OR

Find the value of λ if the four points $A(3, 2, 1), B(4, \lambda, 5), C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.

30. Using properties of determinants, prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

31. Maximize $Z = 3x + 4y$ subject to $x - y \leq -1, -x + y \leq 0, x \geq 0, y \geq 0$. (Show that the feasible region is empty or unbounded, then discuss optimality)

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

OR

Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$.

33. The sum of three numbers is 6. If we multiply the third number by 3 and add the second number to it, we get 11. By adding the first and third numbers, we get double the second number. Represent this algebraically and solve it using the matrix method.

34. Show that a right circular cylinder, which is open at the top and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

OR

Evaluate $\int \frac{x^2+x+1}{(x+1)^2(x+2)}\, dx$.

35. Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Economics of Production**

The total revenue function $R(x)$ for a certain product is given by $R(x) = 13x^2 + 26x + 15$. Marginal Revenue (MR) is defined as the rate of change of total revenue with respect to the number of units sold, x.

Based on the given information, answer the following questions:

(a) Find the Marginal Revenue (MR) function. (1 Mark)

(b) Calculate the Marginal Revenue when 10 units are sold. (3 Marks)

OR

(c) Find the value of x for which the Marginal Revenue is 100. (3 Marks)

37. **Case Study 2: Medical Diagnosis and Conditional Probability**

In a town, a medical test is conducted. It is known that 20% of the population suffers from a certain lung infection. A test is available; for a person who has the infection, the test gives a positive result 90% of the time. For a person who does not have the infection, the test gives a positive result 30% of the time (false positive).

Based on the given information, answer the following questions:

(a) Find the probability that a person is infected and the test result is negative. (1 Mark)

(b) Find the probability that a randomly chosen person gets a positive test result. (3 Marks)

OR

(c) If a person's test result is negative, find the probability that they actually have the infection. (3 Marks)

38. **Case Study 3: Alignment of Satellite Dishes**

Three positions A, B, C in space are used to model the alignment of three satellite dishes. Their coordinates are $A(1, 0, 0), B(0, 1, 0), C(0, 0, 1)$. The plane passing through these three points determines the optimal reception area.

Based on the given information, answer the following questions:

(a) Write the equation of the plane passing through A, B, and C in the intercept form. (1 Mark)

(b) Convert the equation of the plane into the normal form (vector equation $\vec{r} \cdot \hat{n} = d$). (3 Marks)

OR

(c) Find the angle between the optimal reception plane and the xy-plane ($z = 0$). (3 Marks)