PRACTICE QUESTION PAPER - VI (Corrected)

CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. The relation R in the set of natural numbers $\mathbb{N}$ defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$ is:
   (a) Reflexive
   (b) Symmetric
   (c) Transitive
   (d) None of the above
2. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, then $g \circ f(x)$ is:
   (a) $2x$
   (b) $8x$
   (c) $(8x)^{1/3}$
   (d) $2x^3$
3. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is:
   (a) $-\frac{\pi}{3}$
   (b) $\frac{2\pi}{3}$
   (c) $\frac{\pi}{3}$
   (d) $\frac{5\pi}{6}$
4. If $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 3x - 4$, then $f^{-1}(x)$ is:
   (a) $\frac{x}{3} + 4$
   (b) $\frac{x+4}{3}$
   (c) $3x + 4$
   (d) $\frac{1}{3x-4}$

5. The value of $\tan^{-1}(2) + \tan^{-1}(3)$ is:
   (a) $\frac{\pi}{4}$
   (b) $\frac{3\pi}{4}$
   (c) $-\frac{\pi}{4}$
   (d) π
6. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then A^n is equal to:
   (a) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$
   (b) $\begin{bmatrix} 2n+1 & -4n \\ n & 1-2n \end{bmatrix}$
   (c) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$
   (d) $\begin{bmatrix} 2+n & -4-n \\ n & -1-n \end{bmatrix}$
7. If A is a square matrix of order 3 such that $A(adjA) = 10I$, then $|adjA|$ is:
   (a) 1
   (b) 10
   (c) 100
   (d) 1000
8. For what value of k is the matrix $\begin{bmatrix} 2 & k \\ 3 & 1 \end{bmatrix}$ singular?
   (a) 3
   (b) 2/3
   (c) 3/2
   (d) 1
9. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:
   (a) Symmetric matrix
   (b) Skew-symmetric matrix
   (c) Zero matrix
   (d) Identity matrix
10. If A is an $m \times n$ matrix and B is a matrix such that AB and BA are both defined, then the order of B is:
   (a) $m \times n$
   (b) $n \times m$
   (c) $n \times n$
   (d) $m \times m$
11. The interval in which the function $f(x) = \sin x$ is concave up is:
   (a) $(0, \pi)$
   (b) $(\pi, 2\pi)$
   (c) $\left(0, \frac{\pi}{2}\right)$

(d) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

12. The value of $\int \sqrt{1+\sin x}\, dx$ is:

    (a) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

    (b) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$

    (c) $\sin\frac{x}{2} - \cos\frac{x}{2} + C$

    (d) $-\sin x + \cos x + C$

13. The value of c in Rolle's theorem for the function $f(x) = x^2 - 4x + 3$ on the interval $[1, 3]$ is:

    (a) 1

    (b) 2

    (c) 3

    (d) 0

14. The order and degree of the differential equation $y''' + x^2y'' + 2y' = e^x$ are, respectively:

    (a) $3, 1$

    (b) $2, 3$

    (c) $1, 3$

    (d) $3, 3$

15. The area bounded by the parabola $y = 2x - x^2$ and the x-axis is:

    (a) $\frac{1}{3}$ sq. units

    (b) $\frac{2}{3}$ sq. units

    (c) $\frac{4}{3}$ sq. units

    (d) 1 sq. unit

16. If $y = \log_e\left(\frac{x}{e^x}\right)$, then $\frac{dy}{dx}$ is:

    (a) $\frac{1}{x} - e^{-x}$

    (b) $\frac{1}{x} - 1$

    (c) $\frac{1}{x} + 1$

    (d) $\log x - x$

17. The direction cosines of the z-axis are:

    (a) $(1, 0, 0)$

    (b) $(0, 1, 0)$

    (c) $(0, 0, 1)$

    (d) $(1, 1, 1)$

18. The value of λ for which the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \lambda\hat{j} + 3\hat{k}$ are orthogonal is:

    (a) 4

    (b) 3

    (c) 2

    (d) -4

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The function $f(x) = e^x$ is continuous and differentiable everywhere. **Reason (R):** A function is differentiable at a point if and only if it is continuous at that point.

20. **Assertion (A):** The region represented by the linear inequalities in a LPP is called the feasible region. **Reason (R):** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called the optimal solution.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find the value of $\frac{dy}{dx}$ if $x^y = y^x$.

22. Show that the plane $x + 2y - 2z = 9$ is parallel to the line $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z+4}{2}$.

OR

If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

23. Evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}\, dx$.

OR

Evaluate $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$.

24. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, find A^{-1} and verify $AA^{-1} = I$.

25. Two cards are drawn simultaneously (or successively without replacement) from a well-shuffled pack of 52 cards. Find the probability of getting two aces.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Let R be a relation defined by $R = \{(a, b) : a$ and b are both odd or both even$\}$. Show that R is an equivalence relation.

27. Find the equation of the tangent and normal to the curve $y = x^3$ at the point where x coordinate is 1.

OR

Evaluate $\int \frac{\cos x}{(2+\sin x)(3+\sin x)}\, dx$.

28. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

Find the particular solution of $x\frac{dy}{dx} - y + x\sin(\frac{y}{x}) = 0$, given that $y(\pi/2) = \pi/4$.

29. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

OR

Find the vector equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

30. If $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ is an orthogonal matrix, find the values of $x, y,$ and z. (An orthogonal matrix is one whose inverse equals its transpose, i.e., $AA^T = I$).

31. A toy company manufactures two types of toys, A and B. It takes 2 hours to make toy A and 1 hour to make toy B. The company has a maximum of 40 hours available daily. The demand for toy B is limited to 15 units per day. Formulate the LPP to maximize the profit $Z = 5x + 3y$, where x and y are the number of toys A and B respectively. (Formulation only).

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

OR

Evaluate $\int_0^\pi \frac{x}{1+\sin x}\, dx$.

34. Use the matrix method to solve the following system of equations:

$$3x + 4y + 7z = 14$$
$$2x - y + 3z = 4$$
$$x + 2y - 3z = 0$$

35. Show that the semi-vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}(\sqrt{2})$.

OR

Evaluate $\int \frac{x^2}{(x\sin x + \cos x)^2}\, dx$.

36. Find the coordinates of the image of the point $(1, 2, 3)$ in the plane $x + 2y + 4z = 38$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Force, Torque, and Cross Product**

A force $\vec{F} = 3\hat{i} + \hat{j} - 5\hat{k}$ is applied at the point $P(2, -1, 3)$. The torque $(\vec{\tau})$ of this force about the origin O is given by $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the position vector of the point P.

Based on the given information, answer the following questions:

(a) Write the position vector $\vec{r}$ of the point P. (1 Mark)

(b) Find the vector torque $\vec{\tau}$ of the force about the origin O. (2 Marks)

OR

(c) Find the magnitude of the torque $|\vec{\tau}|$. (2 Marks)

38. **Case Study 2: Bayes' Theorem and Diagnostic Testing**

In a certain population, 10% of the people have a certain disease. A diagnostic test is available, and its accuracy is as follows:

- If a person has the disease, the test is positive 90% of the time.

- If a person does not have the disease, the test is negative 80% of the time (i.e., false positive rate is 20%).

Let D be the event of having the disease, and T be the event that the test is positive.

Based on the given information, answer the following questions:

(a) Write the value of $P(T|D')$ (Probability of false positive). (1 Mark)

(b) Find the probability that the test is positive, $P(T)$. (3 Marks)

OR

(c) If a person tests positive, find the probability that they actually have the disease, $P(D|T)$. (3 Marks)

39. **Case Study 3: Rate of Change and Differentiability**

The volume of water in a reservoir, in cubic meters, is modeled by the function $V(t) = \frac{1}{3}t^3 - 4t^2 + 16t + 5$, where t is the time in days $(0 \le t \le 10)$. The rate of change of volume is given by $V'(t)$.

Based on the given information, answer the following questions:

(a) Find the rate of change function $V'(t)$. (1 Mark)

(b) Find the time t when the rate of change of the volume is minimum. (3 Marks)

OR

(c) Find the rate of change of volume after 1 day $(t = 1)$. (3 Marks)