PRACTICE QUESTION PAPER - IX
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = x^2$. Then f is:
   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Neither one-one nor onto
   (d) Both one-one and onto
2. If $f(x) = \frac{x-1}{x+1}$, then $f(f(x))$ is equal to:
   (a) x
   (b) $\frac{1}{x}$
   (c) $-\frac{1}{x}$
   (d) $-x$
3. The value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is:
   (a) $\frac{1}{2}$
   (b) $\frac{\sqrt{3}}{2}$
   (c) 1
   (d) -1
4. The range of $\sec^{-1} x$ is:
   (a) $[0, \pi] - \{\frac{\pi}{2}\}$
   (b) $[-\frac{\pi}{2}, \frac{\pi}{2}]$
   (c) $(0, \pi)$

(d) $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

5. If $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$, then A is a:
   (a) Symmetric matrix
   (b) Diagonal matrix
   (c) Skew-symmetric matrix
   (d) Identity matrix
6. If A is a non-singular matrix of order 3, and $|A| = -4$, then $|adj(A^{-1})|$ is:
   (a) $\frac{1}{4}$
   (b) $-\frac{1}{4}$
   (c) 4
   (d) -4
7. If A is a square matrix such that $|A| = 2$, then $|A \cdot A^T|$ is:
   (a) 2
   (b) 4
   (c) 8
   (d) 16
8. If x, y, z are all different from zero, and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then xyz is equal to:
   (a) -1
   (b) 1
   (c) 0
   (d) 2
9. If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is:
   (a) $e^x \tan e^x$
   (b) $-e^x \tan e^x$
   (c) $e^x \cot e^x$
   (d) $-e^x \cot e^x$
10. The rate of change of the area of a circle with respect to its radius r when $r = 5$ cm is:
    (a) 5π cm^2/cm
    (b) 10π cm^2/cm
    (c) 25π cm^2/cm
    (d) 2π cm^2/cm
11. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to:
    (a) $\tan x + \cot x + C$
    (b) $\tan x - \cot x + C$
    (c) $\sec x - \csc x + C$
    (d) $\tan x \cot x + C$

12. The value of $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\, dx$ is:

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) π

(d) 0

13. The number of arbitrary constants in the particular solution of a differential equation of order 3 is:

(a) 3

(b) 1

(c) 0

(d) 2

14. If $|\vec{a}| = 10, |\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$, then $\vec{a} \cdot \vec{b}$ is equal to:

(a) 12

(b) 16

(c) 20

(d) 10

15. The scalar component of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ on the vector $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ is:

(a) $\frac{9}{\sqrt{3}}$

(b) $\frac{9}{3}$

(c) 3

(d) 9

16. The direction ratios of the normal to the plane $2x - 3y + 4z = 6$ are:

(a) $(\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}})$

(b) $(2, 3, 4)$

(c) $(2, -3, 4)$

(d) $(\frac{1}{2}, -\frac{1}{3}, \frac{1}{4})$

17. The shortest distance between two parallel lines is:

(a) 0

(b) $\frac{|\vec{a_2} - \vec{a_1}|}{|\vec{b}|}$

(c) $\frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$

(d) $\frac{|(\vec{a_2} - \vec{a_1}) \cdot \vec{b}|}{|\vec{b}|}$

18. The corner points of the feasible region determined by the constraints $x + y \leq 2, x \geq 0, y \geq 0$ are:

(a) $(0, 0), (2, 0), (0, 2), (1, 1)$

(b) $(0, 0), (2, 0), (0, 2)$

(c) $(2, 0), (0, 2)$

(d) $(1, 1)$

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.25$, then A and B are independent events. **Reason (R):** Two events A and B are independent if $P(A \cap B) = P(A) \cdot P(B)$.

20. **Assertion (A):** $\int_0^{\pi/2} \log(\tan x)\, dx = 0$. **Reason (R):** $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, where $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

22. Find the area of the triangle having vertices $A(1,1,1), B(1,2,3)$ and $C(2,3,1)$.

OR

Find the direction cosines of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and hence show that the sum of the squares of its direction cosines is 1.

23. Find the value of $\int \frac{e^x(x-3)}{(x-1)^3}\, dx$.

OR

Find the slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$.

24. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

25. A card is drawn from a well-shuffled pack of 52 cards. What is the probability that it is a diamond or a king?

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

27. Evaluate $\int \frac{2x^2+1}{x^2(x^2+4)}\, dx$.

OR

Evaluate $\int \frac{x^2+x}{x^4-1}\, dx$.

28. Find the general solution of the differential equation $x\frac{dy}{dx} - y = x + 2$.

OR

Find the particular solution of $\frac{dy}{dx} = 1 + x + y + xy$, given $y(0) = 0$.

29. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 3y + 4z - 12 = 0$.

OR

Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect.

30. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$.

31. Determine the minimum value of $Z = 3x + 2y$ for the following LPP: Minimize $Z = 3x + 2y$ subject to $x + y \geq 8$, $3x + 5y \leq 15$, $x \geq 0, y \geq 0$. (Justification for unboundedness/infeasibility is required).

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.

OR

Evaluate $\int \frac{1}{\sin x - \sin 2x}\, dx$.

34. Solve the following system of linear equations using the matrix method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

35. Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

OR

Evaluate $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$.

36. Find the distance of the point $P(6, 5, 9)$ from the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Production and Profit Maximization**

A company manufactures x units of a product per day. The production cost $C(x)$ and selling price $p(x)$ per unit are given by:

$$C(x) = 500 + 10x + 0.1x^2$$
$$p(x) = 30 - 0.05x$$

The total revenue is $R(x) = x \cdot p(x)$, and profit is $P(x) = R(x) - C(x)$.

Based on the given information, answer the following questions:

(a) Find the profit function $P(x)$. (1 Mark)

(b) Determine the production level x at which the profit is maximized. (2 Marks)

OR

(c) Calculate the maximum profit. (2 Marks)

38. **Case Study 2: Bernoulli Trials and Probability**

A die is thrown 6 times. Getting an odd number is considered a success. Let X be the random variable representing the number of successes.

Based on the given information, answer the following questions:

(a) Identify the type of probability distribution and define the parameters n and p. (1 Mark)

(b) Find the probability of getting at least 5 successes. (3 Marks)

OR

(c) Find the probability of getting at most 2 successes. (3 Marks)

39. **Case Study 3: Vector Application in Geometry**

A tetrahedron has vertices $A(1,1,1), B(2,1,3), C(3,2,2)$ and $D(3,3,4)$. The volume of a tetrahedron with vertices A, B, C, D is given by $\frac{1}{6}|(\vec{AB} \times \vec{AC}) \cdot \vec{AD}|$.

Based on the given information, answer the following questions:

(a) Write the vectors $\vec{AB}$ and $\vec{AC}$. (1 Mark)

(b) Find the scalar triple product $(\vec{AB} \times \vec{AC}) \cdot \vec{AD}$. (3 Marks)

OR

(c) Calculate the volume of the tetrahedron. (3 Marks)