PRACTICE QUESTION PAPER - XI
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. The minimum number of ordered pairs to form a non-empty equivalence relation on set $A = \{1, 2, 3\}$ is:
   (a) 3
   (b) 6
   (c) 9
   (d) 1
2. If $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = |x|$, then f is:
   (a) One-one
   (b) Onto
   (c) Bijective
   (d) Many-one
3. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is:
   (a) $\frac{3\pi}{4}$
   (b) $\frac{\pi}{4}$
   (c) $\frac{5\pi}{4}$
   (d) $\frac{\pi}{2}$
4. The domain of $f(x) = \cos^{-1}(x^2 - 4)$ is:
   (a) $[-1, 1]$
   (b) $[\sqrt{3}, \sqrt{5}]$
   (c) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

(d) $[-5, -3]$

5. The matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix. Then the value of $a + b + c$ is:

(a) 0

(b) 1

(c) 2

(d) -1

6. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(a) A

(b) I

(c) $I - A$

(d) $2A$

7. If A is a square matrix of order 3×3 and $k|A| = |kA|$, then the value of k is:

(a) k

(b) k^2

(c) k^3

(d) $|A|$

8. The area of a triangle with vertices $(0, 0), (4, 0)$ and $(0, 2)$ is:

(a) 8 sq. units

(b) 6 sq. units

(c) 4 sq. units

(d) 2 sq. units

9. If $y = \log(\tan x)$, then $\frac{dy}{dx}$ is:

(a) $\sec x \csc x$

(b) $\tan x \sec x$

(c) $\sec^2 x$

(d) $\cot x$

10. The function $f(x) = x^2 - 2x + 5$ is strictly increasing in the interval:

(a) $(-\infty, 1)$

(b) $(1, \infty)$

(c) $\mathbb{R}$

(d) $(-\infty, \infty)$

11. $\int e^{\log(\sin x)}\, dx$ is equal to:

(a) $\cos x + C$

(b) $-\cos x + C$

(c) $\sin x + C$

(d) $e^{\sin x} + C$

12. The value of $\int \frac{dx}{x^2 - 4x + 4}$ is:

(a) $\log|x - 2| + C$

(b) $\frac{1}{2}\log|x-2|+C$

(c) $-\frac{1}{x-2}+C$

(d) $-\frac{1}{(x-2)^2}+C$

13. The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is:

(a) e^x

(b) $\log x$

(c) x

(d) x^2

14. If $\vec{a}=2\hat{i}-5\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$, then $|\vec{a}+\vec{b}|$ is:

(a) $\sqrt{18}$

(b) $\sqrt{26}$

(c) $\sqrt{34}$

(d) $\sqrt{42}$

15. The position vector of the midpoint of the vector joining the points $P(2,3,4)$ and $Q(4,1,-2)$ is:

(a) $3\hat{i}+2\hat{j}+\hat{k}$

(b) $6\hat{i}+4\hat{j}+2\hat{k}$

(c) $\hat{i}+\hat{j}+3\hat{k}$

(d) $\hat{i}+2\hat{j}+3\hat{k}$

16. The projection of the line segment joining the points $(1,0,0)$ and $(4,4,12)$ on the x-axis is:

(a) 3

(b) 4

(c) 12

(d) 13

17. The vector equation of the plane passing through the point (a,b,c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$ is:

(a) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

(b) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$

(c) $x+y+z=a+b+c$

(d) $x+y+z=2$

18. If the constraints of an LPP are $x\geq 0, y\geq 0, x+y\leq 6$ and $x\leq 4$, then the number of corner points of the feasible region is:

(a) 3

(b) 4

(c) 5

(d) 6

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are independent events, then A' and B' are also independent. **Reason (R):** $P(A' \cap B') = 1 - P(A \cup B)$.

20. **Assertion (A):** $\frac{d}{dx}(\sin^2 x) = 2 \sin x \cos x$. **Reason (R):** $\frac{d}{dx}(f(x)^n) = nf(x)^{n-1}f'(x)$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$.

22. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$ are perpendicular.

OR

If the position vectors of A and B are $\vec{a}$ and $\vec{b}$, find the position vector of a point C on BA produced such that $BC = 2BA$.

23. Evaluate $\int \frac{\cos x}{1+\sin^2 x}\, dx$.

OR

If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

24. Show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all $x \in [-1, 1]$.

25. If $P(A) = 0.8$ and $P(B|A) = 0.5$, find $P(A \cap B)$. If A and B are independent, what would $P(B|A)$ be?

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation.

27. Evaluate $\int e^{2x} \sin x\, dx$.

OR

Evaluate $\int \frac{x^2+1}{x(x^2-1)}\, dx$.

28. Find the general solution of the differential equation $\frac{dy}{dx} = y \tan x$.

OR

Find the equation of all lines passing through the origin which are tangent to the circle $x^2 + y^2 - 2x + 4y = 0$.

29. Find the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{1} = \frac{z-3}{4}$.

OR

Find the perpendicular distance of the point $(2, 3, 4)$ from the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

30. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$.

31. Solve the following Linear Programming Problem graphically: Maximize $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Using integration, find the area of the region bounded by $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

OR

Evaluate $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$.

34. Find the product of the matrices $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$. Hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

35. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

OR

Evaluate $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$.

36. Find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$ and parallel to the line $\frac{x+3}{3} = \frac{y-3}{2} = \frac{z-2}{5}$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Navigation and Distance**

A ship is being guided by two lighthouse towers. The coordinates of the top of the two towers are $A(1, 2, 3)$ and $B(3, 5, 7)$. A vector representing the direction from A to B is $\vec{AB}$.

Based on the given information, answer the following questions:

(a) Write the vector $\vec{AB}$. (1 Mark)

(b) Find the direction cosines of the vector $\vec{AB}$. (3 Marks)

OR

(c) Find the projection of $\vec{AB}$ on the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. (3 Marks)

38. **Case Study 2: Quality Control and Binomial Distribution**

A batch of 5 items is produced. The probability of an item being defective is 0.1. Let X be the random variable denoting the number of defective items in the batch.

Based on the given information, answer the following questions:

(a) Find the probability of getting exactly one defective item. (1 Mark)

(b) Find the probability of getting at least two defective items. (3 Marks)

OR

(c) Find the mean of the distribution (Expected number of defective items). (3 Marks)

39. **Case Study 3: Area Under Curve Application**

The velocity of a particle moving in a straight line is given by $v(t) = 3t^2 - 4t + 5$ meters per second, for $0 \leq t \leq 3$. The distance covered by the particle in time T is given by the integral $\int_0^T v(t)\, dt$.

Based on the given information, answer the following questions:

(a) Find the initial velocity of the particle (at $t = 0$). (1 Mark)

(b) Find the total distance covered by the particle in the first 2 seconds (from $t = 0$ to $t = 2$). (3 Marks)

OR

(c) Find the distance covered by the particle between $t = 1$ and $t = 3$ seconds. (3 Marks)