PRACTICE QUESTION PAPER - X
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. The binary operation $*$ defined on $\mathbb{Q} - \{-1\}$ by $a * b = a + b + ab$ is:
   (a) Commutative only
   (b) Associative only
   (c) Both commutative and associative
   (d) Neither commutative nor associative
2. The function $f(x) = \sin x$ for all $x \in [0, 2\pi]$ is:
   (a) One-one
   (b) Onto
   (c) Bijective
   (d) Neither one-one nor onto
3. The value of $\cos^{-1}(\cos(7\pi/6))$ is:
   (a) $7\pi/6$
   (b) $5\pi/6$
   (c) $\pi/6$
   (d) $-\pi/6$
4. If $\sin^{-1} x = y$, then:
   (a) $0 \leq y \leq \pi$
   (b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
   (c) $0 < y < \pi$
   (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

5. If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to:
   (a) $\det(A)$
   (b) $\frac{1}{\det(A)}$
   (c) 1
   (d) 0
6. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is:
   (a) 1
   (b) 2
   (c) 3
   (d) -1
7. If A is a square matrix of order 3, and $|A| = -2$, then the value of $|3A|$ is:
   (a) -6
   (b) -18
   (c) -54
   (d) 54
8. If A is a matrix of order $m \times n$, and B is a matrix such that BA is defined, then the order of B is:
   (a) $m \times n$
   (b) $n \times m$
   (c) $m \times k$
   (d) $k \times m$
9. If $x = at^2$ and $y = 2at$, then $\frac{d^2y}{dx^2}$ is:
   (a) $\frac{1}{at^2}$
   (b) $-\frac{1}{at^2}$
   (c) $-\frac{1}{2at^3}$
   (d) $\frac{1}{2at^3}$
10. The tangent to the curve $y = x^3 + 1$ at $(1, 2)$ is parallel to the line:
    (a) $3x - y = 0$
    (b) $3x + y = 0$
    (c) $x - 3y = 0$
    (d) $x + 3y = 0$
11. $\int \frac{1}{\sqrt{9-25x^2}}\, dx$ is equal to:
    (a) $\frac{1}{5}\sin^{-1}(\frac{5x}{3}) + C$
    (b) $\frac{1}{3}\sin^{-1}(\frac{5x}{3}) + C$
    (c) $\sin^{-1}(\frac{5x}{3}) + C$
    (d) $\frac{1}{5}\sin^{-1}(\frac{3x}{5}) + C$
12. The value of $\int_0^{\pi/2} \sin^3 x\, dx$ is:
    (a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 1

(d) 0

13. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

(a) $e^x + e^y = C$

(b) $e^x - e^y = C$

(c) $e^x + e^{-y} = C$

(d) $e^{-x} + e^y = C$

14. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \lambda\hat{j} + 3\hat{k}$ are perpendicular, then the value of λ is:

(a) 5

(b) 8

(c) -8

(d) -5

15. The area of the parallelogram whose diagonals are $\vec{d_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{d_2} = 3\hat{i} + 4\hat{j} - \hat{k}$ is:

(a) $\frac{1}{2}\sqrt{74}$ sq. units

(b) $\sqrt{74}$ sq. units

(c) $\frac{1}{2}\sqrt{84}$ sq. units

(d) 74 sq. units

16. The cosine of the angle between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ is:

(a) $\frac{10}{\sqrt{14}}$

(b) $\frac{20}{\sqrt{14}\sqrt{29}}$

(c) $\frac{20}{14 \cdot 29}$

(d) $\frac{20}{\sqrt{406}}$

17. The equation of the plane parallel to the xy-plane and passing through $(1, 2, 3)$ is:

(a) $x = 1$

(b) $y = 2$

(c) $z = 3$

(d) $x + y = 3$

18. The optimal solution of a LPP occurs at:

(a) Any point in the feasible region

(b) An interior point of the feasible region

(c) A corner point of the feasible region

(d) None of the above

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are independent events, then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. **Reason (R):** For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

20. **Assertion (A):** The function $f(x) = x^3 - 6x^2 + 5$ has local extremum at $x = 0$. **Reason (R):** A function has a local extremum at a point where its derivative is zero or undefined.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = 3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

OR

Show that the three vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $-\hat{i} - 5\hat{j} + 7\hat{k}$ are coplanar.

23. Evaluate $\int \frac{1}{\sqrt{7-6x-x^2}}\, dx$.

OR

Find the slope of the tangent to the curve $x = a\sin^3 t, y = a\cos^3 t$ at $t = \frac{\pi}{4}$.

24. Given $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Find $(adj A)^{-1}$.

25. A family has two children. Find the probability that both children are boys, given that at least one of them is a boy.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x^2 - 5$, show that f is neither one-one nor onto.

27. Evaluate $\int \frac{x^2}{(x+1)(x-2)}\, dx$.

OR

Evaluate $\int \frac{dx}{5-4\cos x}$.

28. Find the particular solution of the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y(\pi/2) = 2$.

OR

Find the points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to the x-axis.

29. Find the equation of the plane that contains the line of intersection of the planes $x+2y+3z-4 = 0$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$.

OR

Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular.

30. Using properties of determinants, prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$.

31. A cooperative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y are estimated to be Rs 10,500 and Rs 9,000 per hectare, respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at the rate of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used. Formulate the problem as an LPP to maximize the total profit.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Find the area of the region bounded by the curves $y^2 = 4x$ and $x^2 = 4y$.

OR

Evaluate $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\, dx$.

34. The sum of three numbers is 2. If the second number is added to the sum of the first and the third, the result is 1. If the third number is subtracted from the sum of the first two, the result is 3. Use the matrix method to find the three numbers.

35. An open tank with a square base and vertical sides is to be constructed to hold a given quantity of water. Show that the cost of lining the tank with lead will be least if the depth is half of the width.

OR

Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)}\, dx$.

36. Find the coordinates of the point where the line through $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the xy-plane.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Economics and Average Cost**

The cost function $C(x)$ for producing x units of a commodity is given by $C(x) = 100 + 7x + 0.05x^2$. The Average Cost (AC) is defined as $AC(x) = \frac{C(x)}{x}$.

Based on the given information, answer the following questions:

(a) Find the Average Cost function $AC(x)$. (1 Mark)

(b) Find the level of output x at which Average Cost is minimum. (3 Marks)

OR

(c) Calculate the minimum Average Cost. (3 Marks)

38. **Case Study 2: Machine Operation and Conditional Probability**

In a factory, machine A produces 30% of the total output, machine B produces 25%, and machine C produces 45%. The past record shows that 1%, 2%, and 2% of the items produced by machines A, B, and C, respectively, are defective. An item is selected at random from the total output.

Based on the given information, answer the following questions:

(a) What is the probability that the selected item was produced by machine A and is defective? (1 Mark)

(b) Find the total probability that the selected item is defective. (3 Marks)

OR

(c) If the selected item is found to be defective, find the probability that it was produced by machine C. (3 Marks)

39. **Case Study 3: Vector Application in Projectiles**

A projectile is launched with initial velocity $\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$ from a point A. The gravitational force is $\vec{g} = -10\hat{k}$. The work done W by the gravitational force in moving the projectile from A to

another point B is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement vector $\vec{AB}$. Assume B has coordinates $(3, 5, 2)$.

Based on the given information, answer the following questions:

(a) Write the force vector $\vec{F}$ due to gravity on a unit mass. (1 Mark)

(b) Assuming the starting point A is the origin $(0, 0, 0)$, find the displacement vector $\vec{AB}$. (3 Marks)

OR

(c) Calculate the work done by the gravitational force in moving the projectile from A to B. (3 Marks)