PRACTICE QUESTION PAPER - VII
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. Let R be a relation on the set $\mathbb{Z}$ of all integers defined by $R = \{(x, y) : x - y \text{ is divisible by } n\}$. R is an equivalence relation for any fixed integer $n \geq 1$. If $n = 5$, the equivalence class of 2, denoted by $[2]$, is:
   (a) $\{\ldots, -3, 2, 7, 12, \ldots\}$
   (b) $\{\ldots, -2, 3, 8, 13, \ldots\}$
   (c) $\{0, 1, 2, 3, 4\}$
   (d) $\{2, 7, 12, 17\}$
2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 3x + 2$, then $f(f(x))$ is:
   (a) $x^4 - 6x^3 + 10x^2 - 3x$
   (b) $x^4 - 6x^3 + 10x^2 - 3x + 2$
   (c) $x^4 - 6x^3 + 13x^2 - 15x + 6$
   (d) $x^4 - 6x^3 + 13x^2 - 15x + 8$
3. The domain of the function $f(x) = \sin^{-1}(2x - 1)$ is:
   (a) $[0, 1]$
   (b) $[-1, 1]$
   (c) $(-1, 1)$
   (d) $(0, 1)$
4. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)\right]$ is:
   (a) $\frac{1}{2}$
   (b) 1
   (c) $\frac{2}{3}$

(d) $\frac{1}{3}$

5. The number of one-one functions from a set A with m elements to a set B with n elements, where $m > n$, is:
   (a) 0
   (b) $n!/(n-m)!$
   (c) n^m
   (d) m^n
6. If A is a 3×3 matrix such that $|A| = 4$, then $|2 \cdot adj(A)|$ is:
   (a) 8
   (b) 16
   (c) 32
   (d) 64
7. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, then $(AB)^T$ is:
   (a) $\begin{bmatrix} -1 & 14 \\ 6 & 13 \end{bmatrix}$
   (b) $\begin{bmatrix} -1 & 6 \\ 14 & 13 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & -1 \\ 6 & 13 \end{bmatrix}$
   (d) $\begin{bmatrix} -1 & 6 \\ 14 & -13 \end{bmatrix}$
8. The area of a triangle with vertices $(1, 0), (6, 0)$ and $(4, 3)$ using determinants is:
   (a) 9 sq. units
   (b) 12 sq. units
   (c) 7.5 sq. units
   (d) 8 sq. units
9. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $(aI + bA)^3$ is equal to:
   (a) $a^3I + b^3A$
   (b) $a^3I + 3a^2bA$
   (c) $a^3I + 3ab^2A$
   (d) $a^3I + b^3I$
10. If the matrix $A = \begin{bmatrix} a & 2 \\ 1 & 4 \end{bmatrix}$ is such that $AA^T = 9I$, then the value of a^2 is:
    (a) 4
    (b) 5
    (c) 1
    (d) 9
11. The maximum value of $f(x) = x^3 - 12x + 1$ in the interval $[-1, 3]$ is:
    (a) 17

(b) 1

(c) -15

(d) -8

12. The value of $\int_0^{\pi/2} \sin 2x \log(\tan x)\, dx$ is:

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) 0

13. If $y = \sec^{-1}(\frac{\sqrt{x}+1}{\sqrt{x}-1}) + \sin^{-1}(\frac{\sqrt{x}-1}{\sqrt{x}+1})$, then $\frac{dy}{dx}$ is:

(a) 1

(b) -1

(c) π

(d) 0

14. The integrating factor of the differential equation $(1 + y^2)dx - (\tan^{-1} y - x)dy = 0$ is:

(a) $e^{\tan^{-1} y}$

(b) $e^{-\tan^{-1} y}$

(c) $\tan^{-1} y$

(d) $\frac{1}{1+y^2}$

15. The distance of the point $P(1, 2, 3)$ from the x-axis is:

(a) 3

(b) $\sqrt{5}$

(c) $\sqrt{10}$

(d) $\sqrt{13}$

16. The equation of the plane passing through the points $(a, 0, 0), (0, b, 0)$ and $(0, 0, c)$ is:

(a) $ax + by + cz = 1$

(b) $ax + by + cz = abc$

(c) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(d) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = abc$

17. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then $|\vec{x}|$ is:

(a) 3

(b) $\sqrt{7}$

(c) 8

(d) 9

18. The projection of the vector $\hat{i} - \hat{j} + \hat{k}$ on the vector $\hat{i} + \hat{j} + \hat{k}$ is:

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{3}$

(c) $\sqrt{3}$

(d) 3

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** $\int \frac{1}{x(1+\log x)}\, dx = \log|1+\log x| + C$. **Reason (R):** The derivative of $\log|f(x)|$ is $\frac{f'(x)}{f(x)}$.

20. **Assertion (A):** If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, then A and B are independent events. **Reason (R):** Two events A and B are independent if $P(A \cap B) = P(A) + P(B)$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find the value of $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

22. Find the magnitude of $\vec{a} \times \vec{b}$, if $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$.

OR

Find the direction cosines of the vector $\vec{r} = 6\hat{i} + 2\hat{j} - 3\hat{k}$.

23. Find the value of $\int_0^1 xe^x\, dx$.

OR

Show that the function $f(x) = |x-1|, x \in \mathbb{R}$ is continuous but not differentiable at $x = 1$.

24. Prove that the function $f : [2, \infty) \to \mathbb{R}$ defined by $f(x) = x^2 - 4x + 5$ is one-one.

25. A pair of dice is thrown. If the two numbers appearing are different, find the probability that the sum of the numbers is 4.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Solve: $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

27. Evaluate $\int \frac{x^2}{x^4+x^2+1}\, dx$.

OR

Evaluate $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx$.

28. Find the general solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$.

OR

Find the equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x-3}, x \neq 3$.

29. Find the value of λ for which the four points $A(4,5,1), B(0,-1,-1), C(3,9,4)$ and $D(-4,4,4)$ are coplanar.

OR

Find the image of the point $(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

30. Using elementary column operations, find the inverse of the matrix $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$.

31. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ subject to $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

OR

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$.

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations:

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

34. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

OR

Evaluate $\int \frac{x^2+x+1}{(x+1)^2(x+2)}\, dx$.

35. Find the equation of the plane passing through the point $(1, 1, 1)$ and containing the line of intersection of the planes $x + 2y - z = 1$ and $3x - y + 4z = 3$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Production Cost and Marginal Cost**

A company manufactures electronic components. The total cost function for producing x units is given by $C(x) = \frac{x^3}{3} - 10x^2 + 100x + 50$. The Marginal Cost (MC) is the rate of change of the total cost with respect to the output, $MC = \frac{dC}{dx}$.

Based on the given information, answer the following questions:

(a) Find the Marginal Cost function $MC(x)$. (1 Mark)

(b) Find the value of x at which Marginal Cost is minimum. (3 Marks)

OR

(c) Calculate the minimum Marginal Cost. (3 Marks)

37. **Case Study 2: Selection based on Urns**

An urn A contains 2 white and 4 black balls. An urn B contains 5 white and 3 black balls. A ball is transferred from urn A to urn B. Then a ball is drawn from urn B.

Based on the given information, answer the following questions:

(a) Find the probability that a black ball is transferred from A to B. (1 Mark)

(b) What is the probability that the ball drawn from B is white? (3 Marks)

OR

(c) If the ball drawn from B is white, what is the probability that a white ball was transferred from A to B? (3 Marks)

38. **Case Study 3: Traffic Monitoring Drones**

Two traffic monitoring drones, D_1 and D_2, are flying along straight-line paths. Their paths are described by the vector equations:

$$D_1 : \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$D_2 : \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

where λ and μ are parameters.

Based on the given information, answer the following questions:

(a) Write the vector parallel to the path of drone D_1. (1 Mark)

(b) Find the angle between the paths of the two drones D_1 and D_2. (3 Marks)

OR

(c) Find the shortest distance between the two drone paths. (3 Marks)