PRACTICE QUESTION PAPER - VIII
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let R be the relation in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : b = a + 1\}$. The relation R is:
   (a) Reflexive
   (b) Symmetric
   (c) Transitive
   (d) None of the above
2. The inverse of the function $f : \mathbb{R} \to (-1, 1)$ defined by $f(x) = \frac{x}{1+|x|}$ is:
   (a) $\frac{y}{1+y}$ for $y \in (-1, 1)$
   (b) $\frac{y}{1-|y|}$ for $y \in (-1, 1)$
   (c) $\frac{1+|y|}{y}$ for $y \in (-1, 1)$
   (d) $\frac{1-|y|}{y}$ for $y \in (-1, 1)$
3. The value of $\cos\left(\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right)$ is:
   (a) 0
   (b) 1
   (c) $\frac{1}{2}$
   (d) -1
4. If $3\tan^{-1} x + \cot^{-1} x = \pi$, then x is equal to:
   (a) 0
   (b) 1

(c) -1

(d) $\frac{1}{2}$

5. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then AA^T is equal to:

   (a) A

   (b) I

   (c) A^T

   (d) O (Zero Matrix)

6. Given $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Then A^3 is:

   (a) $\begin{bmatrix} 3a & 0 \\ 0 & 3b \end{bmatrix}$

   (b) $\begin{bmatrix} a^3 & 0 \\ 0 & b^3 \end{bmatrix}$

   (c) $\begin{bmatrix} a^3 & 3 \\ 3 & b^3 \end{bmatrix}$

   (d) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

7. The determinant of a skew-symmetric matrix of odd order is:

   (a) 1

   (b) -1

   (c) 0

   (d) Always non-zero

8. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} are the cofactors of a_{ij}, then the value of Δ is given by:

   (a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

   (b) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

   (c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

   (d) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

9. If $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is:

   (a) 1

   (b) -1

   (c) 0

   (d) $\frac{1}{2}$

10. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue when $x = 15$ is:

    (a) 116

    (b) 96

    (c) 126

    (d) 36

11. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$ is equal to:

   (a) $\tan x + \cot x + C$

   (b) $\tan x + \csc x + C$

   (c) $\tan x + \sec x + C$

   (d) $\tan x - \cot x + C$

12. The value of $\int_{-1}^{1} |x|\, dx$ is:

   (a) 0

   (b) 1

   (c) 2

   (d) -1

13. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is:

   (a) 1

   (b) 2

   (c) 3

   (d) 4

14. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then $|\vec{a} - \vec{b}|$ is equal to:

   (a) 1

   (b) $\sqrt{5}$

   (c) $\sqrt{13}$

   (d) 5

15. The area of the parallelogram whose adjacent sides are $\hat{\imath} + \hat{k}$ and $2\hat{\imath} + \hat{\jmath} + \hat{k}$ is:

   (a) $\sqrt{3}$ sq. units

   (b) $\sqrt{5}$ sq. units

   (c) $\sqrt{6}$ sq. units

   (d) 3 sq. units

16. The angle between the line $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and the plane $3x + 4y + z = 1$ is:

   (a) $\sin^{-1}\left(\frac{2}{3\sqrt{3}}\right)$

   (b) $\cos^{-1}\left(\frac{2}{3\sqrt{3}}\right)$

   (c) $\sin^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

   (d) $\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

17. The distance of the point $(2, 3, 4)$ from the plane $3x + 2y + 6z + 1 = 0$ is:

   (a) 35

   (b) 5

   (c) $\frac{35}{7}$

   (d) 7

18. The feasible region for an LPP is shown in the figure. The maximum value of $Z = x + 3y$ occurs at:

(a) $(0,0)$

(b) $(0,2)$

(c) $(3,1)$

(d) $(1,3)$

(Assume corner points are $(0,0),(0,2),(3,1),(1,3)$ for calculation.)

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are two events such that $P(A) = 0.4$ and $P(B|A) = 0.5$, then $P(A \cap B) = 0.2$. **Reason (R):** $P(B|A) = \frac{P(A\cap B)}{P(B)}$.

20. **Assertion (A):** The function $f(x) = \tan x$ has no local maxima or minima in $(0, \pi)$. **Reason (R):** The derivative $f'(x) = \sec^2 x$ is strictly positive for all $x \in (0, \pi)$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = (\log x)^{\cos x}$.

22. Find the angle between the vectors $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$.

OR

Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{a} + \vec{b}$, where $\vec{b} = \hat{i} + 2\hat{j} + 0\hat{k}$.

23. Evaluate $\int \frac{1}{x^2-6x+13}\,dx$.

OR

Find the value of k so that the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$.

24. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

25. If $P(A) = 0.3, P(B) = 0.6$ and A and B are independent events, find $P(A \cup B)$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x + 3$ is invertible and find f^{-1}.

27. Find the general solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$.

OR

Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$.

28. Prove that $y = Ae^{-3x} + Be^{2x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$.

OR

Find the equation of the tangent line to the curve $y = \sqrt{4x-3} - 1$ which is parallel to the line $2x - y + 3 = 0$.

29. Find the equation of the plane passing through the point $(1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$.

OR

Find the vector equation of the line passing through the points $A(3, -2, -5)$ and $B(3, -2, 6)$.

30. Express the matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

31. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of model B. Formulate the LPP.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$.

OR

Evaluate $\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} \, dx$.

34. Find the matrix X such that $X \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

35. Find the maximum and minimum values of $f(x) = \sin x + \cos x$ in the interval $[0, \pi]$.

OR

Evaluate $\int \frac{1}{\cos(x-a)\cos(x-b)} \, dx$.

36. Find the shortest distance between the lines $\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Ladder Problem and Rate of Change**

A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. Let x be the distance of the base of the ladder from the wall and y be the height of the top of the ladder on the wall.

Based on the given information, answer the following questions:

(a) Write the relation between x and y using the Pythagorean theorem. (1 Mark)

(b) Find the rate at which the height of the ladder on the wall is decreasing when the base is 4 m away from the wall. (3 Marks)

OR

(c) Find the rate at which the area of the triangle formed by the ladder, the wall, and the ground is changing when the base is 3 m away from the wall. (3 Marks)

38. **Case Study 2: Probability Distribution and Expectation**

A random variable X has the following probability distribution:

X	0	1	2	3
$P(X)$	c	$2c$	$2c$	c

Based on the given information, answer the following questions:

(a) Find the value of the constant c. (1 Mark)

(b) Find the mean (Expected value $E(X)$) of the random variable X. (3 Marks)

OR

(c) Find the probability $P(0 < X < 3)$. (3 Marks)

39. **Case Study 3: Plane Geometry and Intersection**

A large room is designed such that the floor, walls, and ceiling form planes. Consider two adjacent walls, represented by the planes:

$$P_1 : x + 2y - z = 5$$

$$P_2 : 2x - y + 3z = 4$$

Based on the given information, answer the following questions:

(a) Find the vector normal to the plane P_1. (1 Mark)

(b) Find the Cartesian equation of the plane passing through the intersection of P_1 and P_2 and through the point $(0, 0, 0)$. (3 Marks)

OR

(c) Find the angle between the two planes P_1 and P_2. (3 Marks)