PRACTICE QUESTION PAPER - XX
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let R be a relation on the set L of all lines in a plane defined by $(L_1, L_2) \in R$ if L_1 is parallel to L_2. R is:
   (a) Reflexive only
   (b) Symmetric only
   (c) Transitive only
   (d) An Equivalence relation
2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x|x|$, then f is:
   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Both one-one and onto
   (d) Neither one-one nor onto
3. The value of $\cos(\sec^{-1} x + \csc^{-1} x)$, where $|x| \geq 1$, is:
   (a) 1
   (b) -1
   (c) $\pi/2$
   (d) 0
4. The domain of the function $f(x) = \sin^{-1} x + \cos x$ is:
   (a) $[-1, 1]$
   (b) $(-\infty, \infty)$
   (c) $[-1, 1] \cup \mathbb{R}$

(d) $(-1, 1)$

5. If A is a matrix of order $m \times n$ and B is a matrix such that AB^T and B^TA are both defined, then the order of B is:

   (a) $m \times n$

   (b) $n \times m$

   (c) $m \times m$

   (d) $n \times n$

6. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then $a + b$ is equal to:

   (a) 1

   (b) -1

   (c) 5

   (d) -5

7. If A is a square matrix of order 3 such that $A(\text{adj } A) = 10I$, then $|A|$ is:

   (a) 10

   (b) 100

   (c) 1000

   (d) 1

8. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ is:

   (a) xy

   (b) $x + y$

   (c) 1

   (d) 0

9. The derivative of $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ with respect to x is:

   (a) $\frac{3}{1+x^2}$

   (b) $\frac{3}{1-x^2}$

   (c) $\frac{1}{1+x^2}$

   (d) $\frac{1}{1-x^2}$

10. The maximum value of the function $f(x) = 3\cos x + 4\sin x + 8$ is:

    (a) 5

    (b) 12

    (c) 13

    (d) 10

11. $\int \frac{1}{\sqrt{x^2+2x+2}}\, dx$ is equal to:

    (a) $\log|x + 1 + \sqrt{x^2 + 2x + 2}| + C$

    (b) $\sin^{-1}(x + 1) + C$

    (c) $\log|x + \sqrt{x^2 + 2x + 2}| + C$

(d) $\sqrt{x^2+2x+2}+C$

12. The value of $\int_0^2 |x-1|\,dx$ is:

    (a) 1

    (b) 0

    (c) 2

    (d) 1/2

13. The integrating factor (IF) of the differential equation $x\frac{dy}{dx} - y = x^2$ is:

    (a) x

    (b) $1/x$

    (c) e^x

    (d) $\log x$

14. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of λ is:

    (a) 8

    (b) 6

    (c) -8

    (d) 4

15. The scalar triple product $[\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}, \hat{i} - 3\hat{j} + 5\hat{k}]$ is:

    (a) 1

    (b) -1

    (c) 0

    (d) 2

16. The vector equation of the plane $2x + 3y - z = 5$ is:

    (a) $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 5$

    (b) $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 1$

    (c) $x\hat{i} + y\hat{j} + z\hat{k} = 5$

    (d) $\vec{r} = 5(2\hat{i} + 3\hat{j} - \hat{k})$

17. The ratio in which the xy-plane divides the line segment joining the points $A(2, 4, 5)$ and $B(3, 5, -4)$ is:

    (a) $4:5$ internally

    (b) $5:4$ internally

    (c) $4:5$ externally

    (d) $5:4$ externally

18. Any point in the feasible region of an LPP is called a/an:

    (a) Optimal solution

    (b) Corner point

    (c) Feasible solution

    (d) Infeasible solution

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are independent events, then $P(A \text{ and } B) = P(A)P(B)$. **Reason (R):** The multiplication theorem of probability states $P(A \cap B) = P(A)P(B|A)$, and if A and B are independent, $P(B|A) = P(B)$.

20. **Assertion (A):** The function $f(x) = \log x$ has neither a maximum nor a minimum value on $(0, \infty)$. **Reason (R):** $f'(x) = 1/x$, which is always positive for $x \in (0, \infty)$, so the function is always increasing.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. If $y = (\sin x)^{\sin x}$, find $\frac{dy}{dx}$.

22. Find the angle between the vectors $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$.

OR

Find the direction cosines of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and hence show that the sum of the squares of the direction cosines is unity.

23. Evaluate $\int \frac{x^2 \tan^{-1} x}{1+x^2}\, dx$.

OR

Find the interval in which the function $f(x) = \sin x + \cos x, 0 < x < 2\pi$ is strictly increasing.

24. Express the matrix $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

25. Given $P(A) = 0.6, P(B) = 0.3$ and $P(A \cap B) = 0.18$. Find $P(A|B)$. Are A and B independent?

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Simplify $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, if $\frac{a}{b}\tan x > -1$.

27. Evaluate $\int \frac{\cos x}{\sqrt{4-\sin^2 x}}\, dx$.

OR

Evaluate $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$.

28. Find the general solution of the differential equation $(x^2 + xy)dy = (x^2 + y^2)dx$.

OR

Find the coordinates of the point on the curve $y = x^2 + 7x + 3$ at which the tangent is parallel to the x-axis.

29. Find the coordinates of the point where the line joining $A(5, 1, 6)$ and $B(3, 4, 1)$ crosses the yz-plane.

OR

If the vertices A, B, C of a triangle are $(1,2,3), (-1,0,0)$ and $(0,1,2)$, find $\angle ABC$.

30. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

31. Solve the following LPP graphically: Minimize $Z = 3x+5y$ subject to $x+3y \geq 3, x+y \geq 2, x, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region bounded by the curves $x^2 = 4y$ and $4y = 8 - x$.

OR

Evaluate $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx$. (Assume the result $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx = -\frac{\pi}{2}\log 2$ for the purpose of the exam and focus on the steps of integration if this identity is used in a specific way).

33. Given $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the system of equations $2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$.

34. Find the dimensions of the rectangle of maximum area that can be inscribed in a circle of radius r.

OR

Evaluate $\int \frac{x^2}{x^4+1}\, dx$.

35. Find the equation of the plane which contains the line of intersection of the planes $x+2y+3z-4 = 0$ and $2x + y - z + 5 = 0$ and whose perpendicular distance from the origin is $\frac{1}{\sqrt{6}}$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Velocity and Displacement**

The velocity of a particle moving along a straight line is given by $v(t) = t^2 - 4t + 3$, where t is the time in seconds.

Based on the given information, answer the following questions:

(a) Find the time intervals when the particle is moving in the positive direction. (1 Mark)

(b) Find the displacement of the particle in the first 3 seconds (from $t = 0$ to $t = 3$). (3 Marks)

OR

(c) Find the total distance travelled by the particle in the first 3 seconds. (3 Marks)

37. **Case Study 2: Committee Selection**

A committee of 4 students is to be randomly selected from 5 boys and 3 girls. Let X be the random variable representing the number of girls in the committee.

Based on the given information, answer the following questions:

(a) State the number of ways to select a committee of 4 students from the 8 students. (1 Mark)

(b) Find the probability distribution of X. (3 Marks)

OR

(c) Find the probability that the committee has exactly 2 girls. (3 Marks)

38. **Case Study 3: Intersecting Flights**

The path of two aircrafts F_1 and F_2 are modelled by the lines:

$$F_1 : \vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$F_2 : \vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

A radar station is located at $R(1, 0, 2)$.

Based on the given information, answer the following questions:

(a) Write the vector representing the direction of F_1. (1 Mark)

(b) Find the point where F_2 intersects the xz-plane ($y = 0$). (3 Marks)

OR

(c) Find the distance of the radar station R from the line F_1. (3 Marks)