PRACTICE QUESTION PAPER - XIX
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let $A = \{1, 2, 3, 4\}$. The relation $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$ on A is:
   (a) Reflexive and Symmetric
   (b) Reflexive and Transitive
   (c) Symmetric and Transitive
   (d) Only Reflexive
2. If $f(x) = \frac{x-1}{x+1}$, then $f(f(x))$ is equal to:
   (a) $-x$
   (b) x
   (c) $1/x$
   (d) $-1/x$
3. The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is:
   (a) π
   (b) $-\pi/3$
   (c) $\pi/3$
   (d) $-\pi/6$
4. If $\tan^{-1} x + \tan^{-1} y = \pi/4$ and $xy < 1$, then $x + y + xy$ is equal to:
   (a) 0
   (b) 1
   (c) 2
   (d) -1

5. If A is a square matrix, then the matrix AA^T is a:
   (a) Skew-symmetric matrix
   (b) Symmetric matrix
   (c) Diagonal matrix
   (d) Zero matrix
6. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:
   (a) Symmetric matrix
   (b) Skew-symmetric matrix
   (c) Zero matrix
   (d) Identity matrix
7. If A is a square matrix of order 3 and k is a scalar, then $|kA|$ is equal to:
   (a) $k|A|$
   (b) $k^2|A|$
   (c) $k^3|A|$
   (d) $3k|A|$
8. The area of a triangle with vertices $(1, 0), (6, 0), (4, 3)$ is:
   (a) 12 sq units
   (b) 6 sq units
   (c) 9 sq units
   (d) 7.5 sq units
9. If $y = \cos^{-1}(\sin x)$, then $\frac{dy}{dx}$ is:
   (a) -1
   (b) 1
   (c) 0
   (d) $\frac{1}{\sqrt{1-\sin^2 x}}$
10. The critical points of the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ are:
   (a) $x = 1$
   (b) $x = 2$
   (c) $x = 1$ and $x = 2$
   (d) $x = 0$
11. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} \, dx$ is equal to:
   (a) $\tan x + C$
   (b) $2\tan x + C$
   (c) $\sec^2 x + C$
   (d) $x \sec^2 x + C$
12. The value of $\int_0^a \frac{f(x)}{f(x)+f(a-x)} \, dx$ is:
   (a) 0
   (b) $a/2$

(c) a

(d) $2a$

13. The order of the differential equation $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 = \sin x$ is:

(a) 1

(b) 2

(c) 3

(d) Not defined

14. The projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$ is:

(a) $\frac{50}{\sqrt{114}}$

(b) $\frac{60}{\sqrt{114}}$

(c) $\frac{60}{114}$

(d) $\frac{50}{114}$

15. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then the vectors $\vec{a}$ and $\vec{b}$ are:

(a) Parallel

(b) Perpendicular

(c) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$

(d) Unit vectors

16. The distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin is:

(a) 6 units

(b) $\frac{6}{\sqrt{29}}$ units

(c) $\frac{6}{\sqrt{13}}$ units

(d) $\sqrt{29}$ units

17. The Cartesian equation of the line passing through the point $(1, 2, 3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$ is:

(a) $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

(b) $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{-2}$

(c) $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+2}{3}$

(d) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

18. The solution set of the inequation $2x + y > 5$ is a:

(a) Closed half plane

(b) Open half plane

(c) Line segment

(d) Line

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are independent events, then A' and B' are also independent. **Reason (R):** If A and B are independent, $P(A \cap B) = P(A)P(B)$.

20. **Assertion (A):** The derivative of $\sin^{-1}(2x)$ is $\frac{2}{\sqrt{1-4x^2}}$. **Reason (R):** $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}$, where u is a function of x.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

22. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are mutually perpendicular.

OR

Find a vector $\vec{c}$ perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

23. Evaluate $\int \frac{e^x}{e^x - 1}\, dx$.

OR

Find the slope of the tangent to the curve $y = \frac{x-1}{x+1}$ at $x = 0$.

24. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then show that $A^T A = I$.

25. Let X be a random variable that assumes values x_1, x_2, x_3, x_4 with probabilities p_1, p_2, p_3, p_4 respectively, where $p_i = \frac{i}{10}$. Find the value of k if $p_4 = k$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + 2$ is one-one.

27. Evaluate $\int \frac{x^2}{(x-1)^3(x+1)}\, dx$.

OR

Evaluate $\int \frac{x}{\sqrt{x^2+4x+1}}\, dx$.

28. Solve the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$.

OR

Find the equation of all lines having slope 2 and being tangent to the curve $y + 2 = \frac{1}{x-3}$.

29. Find the equation of the plane through the point $(3, -2, -4)$ and perpendicular to the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

OR

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, find the value of k.

30. If A and B are non-singular matrices of the same order, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

31. Minimize $Z = x + 2y$ subject to $2x + y \geq 3, x + 2y \geq 6, x \geq 0, y \geq 0$. (Show the feasible region and find the minimum value.)

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.

OR

Evaluate $\int_{-1}^{1} \frac{\sin x}{1+x^2}\, dx$.

33. Solve the following system of equations using matrix inversion: $x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2$.

34. Show that the semi-vertical angle of the cone of maximum volume and given total surface area is $\sin^{-1}(1/3)$.

OR

Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\, dx$.

35. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Volume of a Right Circular Cone**

The volume V of a right circular cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius is increasing at a constant rate of 2 cm/s and the height is decreasing at a rate of 3 cm/s.

Based on the given information, answer the following questions:

(a) Write the expression for the rate of change of volume $\frac{dV}{dt}$. (1 Mark)

(b) Find the rate of change of the volume when $r = 6$ cm and $h = 9$ cm. (3 Marks)

OR

(c) Find the rate of change of the area of the base when $r = 6$ cm. (3 Marks)

37. **Case Study 2: Machine Reliability**

A manufacturing process involves two independent machines, A and B. The probability of machine A failing is $P(A) = 0.1$, and the probability of machine B failing is $P(B) = 0.05$.

Based on the given information, answer the following questions:

(a) Find the probability that machine A works. (1 Mark)

(b) Find the probability that both machines fail. (3 Marks)

OR

(c) Find the probability that at least one machine works. (3 Marks)

38. **Case Study 3: Path of an Object**

An object is moving in space such that its position at time t is given by the line $L : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z-0}{3}$. A detector is placed on the plane $\Pi : x + y + z = 10$.

Based on the given information, answer the following questions:

(a) Write the coordinates of the point on the line L for any parameter λ. (1 Mark)

(b) Find the coordinates of the point where the object intersects the detector plane Π. (3 Marks)

OR

(c) Find the equation of the plane passing through the point $(2, 1, 0)$ and containing the y-axis. (3 Marks)
