# PRACTICE QUESTION PAPER - XII
## CLASS XII - MATHEMATICS (041)

**Time Allowed: 3 Hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

# SECTION A (20 Marks)

*This section comprises* **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

**Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)**

1. If the function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 4$, then $f^{-1}(x)$ is:
   (a) $\frac{x}{3} + 4$
   (b) $4x + 3$
   (c) $\frac{x+4}{3}$
   (d) $\frac{1}{3x-4}$
2. Let $R$ be a relation in the set $L$ of all lines in a plane defined as $R = \{(L_1, L_2) : L_1 \perp L_2\}$. The relation $R$ is:
   (a) Reflexive
   (b) Symmetric
   (c) Transitive
   (d) Equivalence
3. The principal value of $\cot^{-1}(-\sqrt{3})$ is:
   (a) $-\frac{\pi}{6}$
   (b) $\frac{5\pi}{6}$
   (c) $\frac{2\pi}{3}$
   (d) $\frac{\pi}{6}$
4. The value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)\right)$ is:
   (a) $\frac{5}{12}$
   (b) $\frac{1}{12}$
   (c) $\frac{5}{6}$

(d) $\frac{25}{12}$

5. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $A^{-1}$ is:

(a) $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

(d) $\begin{bmatrix} -\cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

6. If $A$ and $B$ are symmetric matrices of the same order, then $AB - BA$ is a:

(a) Symmetric matrix

(b) Skew-symmetric matrix

(c) Zero matrix

(d) Identity matrix

7. If $A$ is an orthogonal matrix of order $n$, then $|A|$ is:

(a) 0

(b) 1

(c) $\pm 1$

(d) $n$

8. If the points $(x, -2), (5, 2)$ and $(8, 8)$ are collinear, then $x$ is equal to:

(a) $-1$

(b) 1

(c) 2

(d) 0

9. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is:

(a) $\frac{1+\log x}{(1-\log x)^2}$

(b) $\frac{\log x}{(1+\log x)^2}$

(c) $\frac{x}{(1+\log x)^2}$

(d) $\frac{1+\log x}{\log x}$

10. The interval in which the function $f(x) = 2x^3 - 3x^2 - 12x + 6$ is strictly decreasing is:

(a) $(-2, 1)$

(b) $(-1, 2)$

(c) $(-\infty, -1)$

(d) $(2, \infty)$

11. $\int \frac{1+\cos x}{x+\sin x}\, dx$ is equal to:

(a) $\log|1 + \cos x| + C$

(b) $\log|x + \sin x| + C$

(c) $\sin x + \cos x + C$

(d) $\log|1+\sin x|+C$

12. The value of $\int_0^1 x(1-x)^n \, dx$ is:
    (a) $\frac{1}{(n+1)(n+2)}$
    (b) $\frac{1}{n+1} - \frac{1}{n+2}$
    (c) $\frac{n}{n+1} - \frac{n}{n+2}$
    (d) $\frac{1}{n+1} + \frac{1}{n+2}$

13. The order of the differential equation of all non-vertical lines in a plane is:
    (a) 1
    (b) 2
    (c) 3
    (d) 0

14. If $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$, then the angle between vectors $\vec{a}$ and $\vec{b}$ is:
    (a) $\pi/4$
    (b) $\pi/2$
    (c) $\pi/3$
    (d) 0

15. The volume of the parallelepiped whose coterminous edges are $\hat{\imath}, \hat{\jmath}, \hat{k}$ is:
    (a) 3
    (b) 1
    (c) 0
    (d) $\sqrt{3}$

16. The equation of the plane $3x - 4y + 12z = 5$ in normal form is:
    (a) $\frac{3}{5}x - \frac{4}{5}y + \frac{12}{5}z = 1$
    (b) $\frac{3}{13}x - \frac{4}{13}y + \frac{12}{13}z = \frac{5}{13}$
    (c) $3x - 4y + 12z = 5$
    (d) $\frac{3}{13}x + \frac{4}{13}y + \frac{12}{13}z = \frac{5}{13}$

17. The distance of the point $(2, 3, 4)$ from the plane $x = 0$ is:
    (a) 2
    (b) 3
    (c) 4
    (d) 5

18. The corner points of the feasible region are $(0, 0), (5, 0), (3, 4), (0, 5)$. If the objective function is $Z = 2x + 3y$, the maximum value of $Z$ is:
    (a) 10
    (b) 15
    (c) 18
    (d) 12

**Assertion-Reasoning Based Questions**

*In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:*

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If $P(A) = 0.4$ and $P(B) = 0.7$, and $A$ and $B$ are mutually exclusive, then $P(A \cup B) = 1.1$. **Reason (R):** For mutually exclusive events $A$ and $B$, $P(A \cup B) = P(A) + P(B)$.

20. **Assertion (A):** $\int_1^e \log x \, dx = 1$. **Reason (R):** The integral of $\log x$ is $x \log x - x + C$.

# SECTION B (10 Marks)

*This section comprises* **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \log\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$.

22. Show that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular.

**OR**

Find the unit vector perpendicular to both vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$.

23. Evaluate $\int \frac{dx}{x\sqrt{x^2-1}}$.

**OR**

The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

24. If $f(x) = \frac{x-1}{x+1}$, verify that $f(f(x)) = -\frac{1}{x}$.

25. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the probability of getting two aces.

# SECTION C (18 Marks)

*This section comprises* **6** *questions of* **3** *marks each.*

26. Prove that $\tan^{-1}(\sqrt{x}) = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in (0, 1)$.

27. Evaluate $\int \frac{\cos x - \sin x}{1+\sin 2x} \, dx$.

**OR**

Evaluate $\int \frac{xe^x}{(x+1)^2} \, dx$.

28. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$.

**OR**

A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 10 cm.

29. Find the coordinates of the point where the line joining $A(5, 1, 6)$ and $B(3, 4, 1)$ crosses the $yz$-plane.

**OR**

Find the value of $p$ for which the plane $x + y + z = 7$ is perpendicular to the plane $2x + py + 2z = 11$.

30. Solve for $x$ if $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$.

31. Find the maximum and minimum values of $Z = 3x + 4y$ subject to the constraints $x + y \leq 4, x \geq 0, y \geq 0$.

## SECTION D (20 Marks)

*This section comprises* **4** *questions of* **5** *marks each.*

33. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.

**OR**

Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$.

34. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find $AB$. Hence, use the result to solve the system of equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

35. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

**OR**

Evaluate $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$.

36. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

## SECTION E (12 Marks)

*This section comprises* **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Production Cost and Marginal Revenue**

A company manufactures electronic gadgets. The demand function for the gadget is $p(x) = 15 - 2x$, where $x$ is the number of units sold (in hundreds) and $p(x)$ is the price per unit (in thousands of rupees). The total revenue $R(x)$ is given by $R(x) = x \cdot p(x)$. Marginal Revenue (MR) is defined as $MR = \frac{dR}{dx}$.

Based on the given information, answer the following questions:

(a) Find the Total Revenue function $R(x)$. (1 Mark)

(b) Find the Marginal Revenue (MR) when $x = 3$. (3 Marks)

**OR**

(c) Find the value of $x$ for which the Marginal Revenue is zero. (3 Marks)

38. **Case Study 2: Independent Events and Conditional Probability**

In a school, 70% of the students pass in Mathematics and 80% of the students pass in Physics. It is known that $A$ (student passes in Maths) and $B$ (student passes in Physics) are independent events.

Based on the given information, answer the following questions:

(a) Find the probability that a student fails in both Mathematics and Physics. (1 Mark)

(b) Find the probability that a student passes in at least one of the subjects. (3 Marks)

**OR**

(c) If a student has passed in Physics, what is the probability that he/she also passed in Mathematics? (3 Marks)

39. **Case Study 3: Line of Sight and Intersection**

A telecommunication company is planning to lay a cable connection. The path of the cable is a straight line, and its vector equation is given by:

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

A major fault line lies on the plane given by the equation $x + y + z = 1$.

Based on the given information, answer the following questions:

(a) Write the Cartesian equation of the line of the cable path. (1 Mark)

(b) Find the point where the cable line intersects the $xy$-plane ($z = 0$). (3 Marks)

**OR**

(c) Find the coordinate of the point where the cable line intersects the fault plane $x + y + z = 1$. (3 Marks)