# PRACTICE QUESTION PAPER - III
## CLASS XII - MATHEMATICS (041)

**Time Allowed: 3 Hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

# SECTION A (20 Marks)

*This section comprises* **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

### Multiple Choice Questions (MCQs)

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3$. Then $f$ is:
   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Neither one-one nor onto
   (d) Both one-one and onto
2. The value of $\sin^{-1}(\cos x)$ where $x \in \left(\frac{\pi}{2}, \pi\right)$ is:
   (a) $x - \frac{\pi}{2}$
   (b) $\frac{\pi}{2} - x$
   (c) $\frac{3\pi}{2} - x$
   (d) $x - \frac{3\pi}{2}$
3. The number of bijective functions from a set $A$ of size $n$ to itself is:
   (a) $n^2$
   (b) $2^n$
   (c) $n!$
   (d) $n^n$
4. The domain of the function $f(x) = \sin^{-1}(2x - 1)$ is:
   (a) $[0, 1]$
   (b) $[-1, 1]$
   (c) $(-1, 1)$
   (d) $[-\pi/2, \pi/2]$

5. If $f(x) = |\cos x|$ and $g(x) = x^2$, then $(g \circ f)\left(\frac{\pi}{4}\right)$ is:
   (a) $1/2$
   (b) $1/\sqrt{2}$
   (c) $1$
   (d) $\pi/2$
6. If $A$ is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to:
   (a) $A$
   (b) $I - A$
   (c) $I + A$
   (d) $3A$
7. For a non-zero scalar $k$, if $A$ is a square matrix of order 3, then $|kA|$ is equal to:
   (a) $k|A|$
   (b) $k^2|A|$
   (c) $k^3|A|$
   (d) $|A|$
8. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A(adj(A))$ is equal to:
   (a) $-2I$
   (b) $2I$
   (c) $I$
   (d) $0$
9. The system of equations $x + y + z = 1$, $2x + 2y + 2z = 2$, and $3x + 3y + 3z = 3$ has:
   (a) A unique solution
   (b) No solution
   (c) Infinitely many solutions
   (d) Exactly two solutions
10. If $A$ is a matrix such that $A \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then the order of matrix $A$ must be:
   (a) $2 \times 2$
   (b) $2 \times 1$
   (c) $1 \times 2$
   (d) $2 \times 3$
11. The derivative of $\log(\sec x + \tan x)$ with respect to $x$ is:
   (a) $\cos x$
   (b) $\tan x$
   (c) $\sec x$
   (d) $\sec x + \tan x$
12. If $y = x^{\sin x}$, then $\frac{dy}{dx}$ is:
   (a) $\sin x \cdot x^{\sin x - 1}$
   (b) $x^{\sin x}(\cos x \log x)$

(c) $x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$

(d) $x^{\sin x}\left(\frac{\sin x}{x}\right)$

13. The value of $\int e^x(\tan x + \sec^2 x)\,dx$ is:

    (a) $e^x \sec x + C$

    (b) $e^x \tan x + C$

    (c) $e^x(\tan x + \sec x) + C$

    (d) $e^x \cot x + C$

14. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

    (a) 3

    (b) $-\frac{1}{3}$

    (c) $-3$

    (d) $\frac{1}{3}$

15. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

    (a) $\tan^{-1} y = \tan^{-1} x + C$

    (b) $\tan^{-1} x = \tan^{-1} y + C$

    (c) $\log(1 + y^2) = \log(1 + x^2) + C$

    (d) $\tan(y) = \tan(x) + C$

16. The area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is:

    (a) 1 sq. unit

    (b) 2 sq. units

    (c) 4 sq. units

    (d) $\pi$ sq. units

17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are:

    (a) Parallel

    (b) Perpendicular

    (c) Inclined at $60°$

    (d) Inclined at $45°$

18. The vector equation of the plane $2x - 3y + 4z = 6$ is:

    (a) $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$

    (b) $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$

    (c) $2\hat{i} - 3\hat{j} + 4\hat{k} = 6$

    (d) $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 6$

**Assertion-Reasoning Based Questions**

*Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:*

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The function $f(x) = \sin x$ has a local maxima at $x = \pi/2$ in $(0, \pi)$. **Reason (R):** For a continuous function, a local maxima occurs at a critical point $c$ if $f'(c) = 0$ or $f'(c)$ is undefined.

20. **Assertion (A):** The optimal solution of a LPP (if it exists) must lie at a corner point of the feasible region. **Reason (R):** The feasible region is formed by a set of linear inequalities, and its vertices correspond to the optimal solutions.

## SECTION B (10 Marks)

*This section comprises **5** questions of **2** marks each.*

21. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$.

22. Find the angle between the two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

**OR**

Find the direction ratios of the line perpendicular to the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

23. Evaluate $\int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}}\, dx$.

**OR**

Evaluate $\int_0^{\pi/4} \tan^2 x \, dx$.

24. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$, find $A^2 - 7A + 22I$.

25. A die is thrown three times. Find the probability of getting exactly one six.

## SECTION C (18 Marks)

*This section comprises **6** questions of **3** marks each.*

26. Show that the relation $R$ in the set $A = \{x \in \mathbb{Z} : 0 \le x \le 12\}$ given by $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is an equivalence relation.

27. Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to the $x$-axis.

**OR**

A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 10 cm.

28. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$.

**OR**

Find the particular solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$, given that $y(1) = 0$.

29. Find the distance of the point $(2, 3, -5)$ from the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) = 9$.

**OR**

Find the value of $\lambda$ such that the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + 7\hat{j} + 3\hat{k}$ are coplanar.

30. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ using Elementary Column Operations.

31. Find the corner points of the feasible region determined by the following constraints: $x + y \le 4$, $2x + y \le 5$, $x \ge 0$, $y \ge 0$.

## SECTION D (20 Marks)

*This section comprises* **4** *questions of* **5** *marks each.*

33. Find the area of the region bounded by $y = x^2$ and $y = |x|$.

**OR**

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

34. Solve the following system of linear equations using the matrix method:

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

35. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

**OR**

Evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$.

36. Find the coordinates of the foot of the perpendicular drawn from the point $A(1, 8, 4)$ to the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{3}$. Also, find the perpendicular distance.

## SECTION E (12 Marks)

*This section comprises* **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Geometric Mean and Dot Product**

In a Physics experiment, forces are applied to a particle. The forces are represented by vectors $\vec{F_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{F_2} = \hat{i} + 2\hat{j} + \hat{k}$. The particle is displaced from point $A(1, 1, 1)$ to $B(2, 3, 5)$.

Based on the given information, answer the following questions:

(a) Find the resultant force vector, $\vec{F} = \vec{F_1} + \vec{F_2}$. (1 Mark)

(b) Find the displacement vector, $\vec{d}$. (1 Mark)

(c) Calculate the work done by the resultant force, given by $W = \vec{F} \cdot \vec{d}$. (2 Marks)

**OR**

(d) Find the vector component of $\vec{F_1}$ along $\vec{F_2}$. (2 Marks)

38. **Case Study 2: Inspection and Conditional Probability**

A manufacturing plant has three machines $M_1, M_2, M_3$. The production rates are $25\%, 35\%, 40\%$ respectively. Past data shows that $2\%, 4\%, 5\%$ of the items produced by $M_1, M_2, M_3$ are defective, respectively. An item is selected at random from the total production.

Based on the given information, answer the following questions:

(a) Find the probability that the item was produced by machine $M_3$. (1 Mark)

(b) Find the probability that the item is defective. (3 Marks)

**OR**

(c) If the item selected is found to be defective, find the probability that it was produced by machine $M_2$. (3 Marks)

39. **Case Study 3: Area Under Curve and Limits**

A physical quantity is modeled by the function $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$. This function describes the shape of a profile section in engineering design over the interval $[0, 2]$.

Based on the given information, answer the following questions:

(a) Check if the function $f(x)$ is continuous at $x = 1$. (1 Mark)

(b) Calculate the total area under the curve $f(x)$ over the interval $[0, 2]$. (3 Marks)

**OR**

(c) Find the value of $\int_0^2 x f(x)\, dx$. (3 Marks)