ISC CLASS XII MATHEMATICS (TEST PAPER 8) - SET 08

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Z}$ defined by $a * b = a + b + ab$. Find the inverse of the element 2. [1]
2. Evaluate: $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$. [1]
3. Determine if the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - 3x$ is one-one. [1]
4. Find $f^{-1}(x)$ if $f(x) = \frac{e^x - e^{-x}}{2}$. [1]
5. Find $\frac{dy}{dx}$ if $y = x^x$. [1]
6. Write the integrating factor (I.F.) of the differential equation: $\frac{dy}{dx} - \frac{y}{x} = x \cos x$. [1]
7. State the value of k that makes $f(x) = \frac{\sin x}{x}$ continuous at $x = 0$. [1]
8. Write the value of the integral $\int_1^3 (2x + 5)dx$ as the limit of a sum. (Write the expression only). [1]
9. If $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.2$. Find $P(A \cup B)$. [1]
10. The total probability of a distribution is $\sum P(X) = k$. What is the value of k? [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. Differentiate $y = \left(\frac{x}{\sin x}\right)^x$ with respect to x. [2]
2. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$ at $x = 10$. [2]
3. Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.7$, and $P(B|A) = 0.5$. Find $P(A \cup B)$. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Verify Lagrange's Mean Value Theorem for the function $f(x) = x^3 - 6x + 5$ in the interval $[1, 3]$. [4]
2. Solve the differential equation: $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$, given $y(1) = \frac{\pi}{4}$. [4]
3. Evaluate: $\int \frac{3x-1}{(x^2+1)(x+1)} dx$. [4]
4. Find the value of x for which the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{pmatrix}$ is singular. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. A wire of length 20 m is to be cut into two pieces. One piece is to be made into a square and the other into an equilateral triangle. How should the wire be cut so that the sum of the areas of the square and the triangle is minimum? [6]
2. Evaluate: $\int \sin(\log x) dx$. [6]
3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + y + z &= 3 \\ x - 2y + 3z &= 2 \\ 2x - y + z &= 6 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that: $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$. [6] **(b)** A coin is tossed 6 times. Find the probability of getting at least 5 successes. [6] **(c)** An urn contains 3 red and 5 black balls. A second urn contains 6 red and 4 black balls. A ball is drawn from the first urn and put into the second urn, and then a ball is drawn from the second urn. Find the probability that the second ball is red. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$, find the magnitude of the projection of $\vec{b}$ on $\vec{a}$. [2]
2. Find the magnitude of $\vec{a} \times \vec{b}$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the angle between the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 5$. [6]
2. Using integration, find the area bounded by the parabolas $y = x^2$ and $x = y^2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The marginal revenue function for a firm is $MR = 50 - 6x - 2x^2$. Find the total revenue function $R(x)$ and the demand function $p(x)$. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 3x + 2y$ Subject to the constraints:

$$x + 2y \leq 10$$
$$3x + y \leq 15$$
$$x, y \geq 0$$

[4]

2. The following regression equations are given: $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$. Find the mean of x and y and the coefficient of correlation r. [6]