ISC CLASS XII MATHEMATICS (TEST PAPER 14) - SET 14

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Q}$ defined by $a * b = a + b + ab$. Find the identity element for $*$. [1]
2. Evaluate: $\tan\left(\cos^{-1}\frac{4}{5}\right)$. [1]
3. State the domain of the function $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$. [1]
4. Let R be a relation on $\mathbb{Z}$ defined by xRy if $x + 2y$ is an integer. Check if R is transitive. [1]
5. Find $\frac{dy}{dx}$ if $y = \cos^{-1}(x^2)$. [1]
6. Find the value of $\int_0^{2\pi} \sin^5 x dx$. [1]
7. Determine if $f(x) = \begin{cases} |x-1| & \text{if } x \leq 2 \\ x^2 - 4 & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$. [1]
8. Write the form of the general solution of the linear differential equation $\frac{dy}{dx} + Py = Q$. [1]
9. If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.6$, find $P(A \cap B)$. [1]
10. A random variable X has values $1, 2, 3$ with probabilities $0.2, 0.5, 0.3$. Find $E(X)$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \sin^2 x$, find $\frac{d^2y}{dx^2}$. [2]
2. The volume of a sphere is increasing at the rate of 3π cm^3/s. Find the rate at which its surface area is increasing when the radius is 1 cm. [2]
3. Three electric bulbs are selected at random from 15 bulbs, 5 of which are defective. Find the probability that none is defective. [2]

Question 3 (4 $\times$ 4 Marks = 16 Marks)

Answer the following questions.

1. Use differentiation to find the approximate value of $\tan(44°)$. (Use $1° \approx 0.0175$ radians). [4]
2. Evaluate: $\int e^x(\sin x + \cos x)dx$. [4]
3. Solve the differential equation: $(x + y)dy = (x - y)dx$. [4]
4. Solve for x in the equation: $\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ b & a & x+c \end{vmatrix} = 0$. [4]

Question 4 (3 $\times$ 6 Marks = 18 Marks)

Answer the following questions.

1. Find the maximum area of a rectangle that can be inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. [6]
2. Evaluate: $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$. [6]
3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Show that the function $f : \mathbb{R} \to (0, \infty)$ defined by $f(x) = 5^{x+2}$ is invertible. Find the inverse function $f^{-1}(x)$. [6] **(b)** An electronic assembly consists of two sub-systems, say A and B. From previous testing procedures, the following probabilities are assumed to be known: $P(A \text{ fails}) = 0.2, P(B \text{ fails alone}) = 0.15, P(A \text{ and } B \text{ fail}) = 0.15$. Find $P(A \text{ fails}|B \text{ has failed})$. [4] **(c)** Three bags contain balls as follows: Bag I: 3 Red, 4 Black; Bag II: 4 Red, 5 Black; Bag III: 5 Red, 3 Black. A bag is chosen at random and a ball is drawn. If the ball is Red, find the probability that it came from Bag I. [5]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the value of λ if the points $A(\lambda, 2, 1)$, $B(4, -1, 3)$, and $C(2, -3, 0)$ are collinear. [2]
2. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the shortest distance between the lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$. [6]
2. Using integration, find the area of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the line $x = 1$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total revenue $R(x)$ and the total cost $C(x)$ for producing x units are given by $R(x) = 13x - 0.01x^2$ and $C(x) = 2x + 100$. Find the level of production x that maximizes the profit. Find the maximum profit. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 5x + 7y$ Subject to the constraints:
$$x + y \leq 4$$
$$3x + 8y \geq 24$$
$$x \geq 0$$
$$y \geq 0$$
[4]

2. The two lines of regression are $y = 0.5x + 5$ and $x = 0.2y + 1$. Find the coefficient of correlation r. If the standard deviation of y is 2, find the standard deviation of x. [6]