ISC CLASS XII MATHEMATICS (TEST PAPER 6) - SET 06

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 $\times$ 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Z}$ defined by $a * b = |a - b|$. Check if $*$ is commutative. [1]
2. Find the principal value of $\sec^{-1}(-2)$. [1]
3. If $f(x) = |x| - 5$ and $g(x) = x^2 + 1$, find $f \circ g(1)$. [1]
4. State the range of the function $f(x) = \frac{\pi}{2} - \cos^{-1} x$. [1]
5. If $x^2 + y^2 = 5$, find $\frac{dy}{dx}$ at $(1, 2)$. [1]
6. Evaluate: $\int e^x(1 + x)dx$. [1]
7. What is the degree of the homogeneous differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$? [1]
8. If $P(A) = 0.4$ and $P(A \cap B) = 0.1$, find $P(A \cap B')$. [1]
9. If the mean of a probability distribution is 10, what is $E(2X + 3)$? [1]
10. If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, find a non-zero row matrix X such that $XA = 0$. [1]

Question 2 (3 $\times$ 2 Marks = 6 Marks)

Answer the following questions.

1. If $x = \sin^3 t$ and $y = \cos^3 t$, find $\frac{dy}{dx}$. [2]
2. A balloon is spherical in shape. Gas is leaking out of the balloon at the rate of 10 cm^3/min. How fast is the radius of the balloon decreasing when the radius is 15 cm? [2]
3. A bag contains 4 red and 5 black balls. Two balls are drawn at random without replacement. Find the probability that both balls are red. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Find a matrix X such that $2A + B + X = 0$, where $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$. [4]
2. Evaluate: $\int \frac{dx}{x^2-6x+13}$. [4]
3. Show that $f(x) = \log(\cos x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$. [4]
4. Solve the differential equation: $\frac{dy}{dx} - y = \cos x - \sin x$. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. An open tank with a square base and vertical sides is to be constructed from a metal sheet of given area A. Show that the cost of material will be least if the depth is half of the width. [6]
2. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$. [6]
3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + 2y + z &= 4 \\ -x + y + z &= 0 \\ x - 3y + z &= 2 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** If $f : \mathbb{R} \to [4, \infty)$ is defined by $f(x) = x^2 + 4$, show that f is not invertible. Restrict the domain to $D = [0, \infty)$ and find f^{-1}. [6] **(b)** Find the mean and variance of the number of doubles when a pair of dice is thrown 3 times. [6] **(c)** In a class, 40% students study Mathematics and 30% study Biology. 10% study both. Find the probability that a student studies Mathematics given he/she studies Biology. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the area of the parallelogram whose diagonals are $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$. [2]
2. Find the scalar product of $(\vec{a} + 3\vec{b})$ and $(2\vec{a} - \vec{b})$, if $|\vec{a}| = 2$, $|\vec{b}| = 3$, and $\vec{a} \cdot \vec{b} = 1$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the distance of the origin from the plane $3x - 4y + 12z = 52$. Find the vector equation of the plane. [6]
2. Using integration, find the area bounded by the parabola $y^2 = 4x$ and the latus rectum. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The demand function for a certain commodity is $p = 100 - 4x$ and the cost function is $C(x) = 50x + 200$. Find the profit function $P(x)$, the marginal profit, and the output level x at which the marginal revenue is zero. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 2x + 3y$ Subject to the constraints:
$$\begin{aligned} x + 2y &\leq 10 \\ 2x + y &\leq 8 \\ x, y &\geq 0 \end{aligned}$$
[4]
2. For a bivariate distribution, the two regression coefficients are $b_{yx} = -0.5$ and $b_{xy} = -0.8$. If the variance of y is 16, find the coefficient of correlation (r) and the standard deviation of x. [6]