ISC CLASS XII MATHEMATICS (TEST PAPER 2)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let R be a relation on the set $\mathbb{Z}$ of integers defined by $(a, b) \in R$ if and only if $a - b$ is divisible by 3. State the number of distinct equivalence classes of R. [1]
2. If A is a square matrix of order 3 and $|A| = -5$, find the value of $|\text{adj}(A^2)|$. [1]
3. Evaluate: $\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$. [1]
4. Find the value of k for which the function $f(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. [1]
5. Find the order and degree of the differential equation: $\left(\frac{d^3y}{dx^3}\right)^2 - 5\frac{d^2y}{dx^2} + \sqrt{\frac{dy}{dx} + 1} = 0$. [1]
6. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$. [1]
7. Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x - 1|$ is neither one-one nor onto. [1]
8. Using the first derivative test, find the local maxima/minima for $f(x) = x(x - 1)^2$. [1]
9. If $P(A) = 0.6$, $P(B) = 0.4$, and $P(A \cap B) = 0.24$. Are A and B independent events? Justify. [1]
10. If X is a random variable that follows a Binomial distribution $B(4, p)$ and $P(X = 0) = \frac{16}{81}$, find the value of p. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = (\cos x)^x + x^y$, find $\frac{dy}{dx}$. [2]
2. Verify Lagrange's Mean Value Theorem for the function $f(x) = x^2 - 4x - 3$ on the interval $[1, 4]$. [2]
3. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the probability that both are kings. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. [4]
2. Find the equation of the tangent and normal to the curve $x^2 + y^2 = 25$ at the point $(3, -4)$. [4]
3. Using the properties of definite integrals, evaluate:

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

   [4]

4. Prove that: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$. [6]
2. Find the particular solution of the differential equation: $(1 + x^2)dy + 2xydx = \cot x dx$, given $y = 0$ when $x = \frac{\pi}{2}$. [6]
3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + y + z &= 6 \\ y + 3z &= 11 \\ x - 2y + z &= 0 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that the relation R on the set $A = \{x \in \mathbb{Z} : 0 \le x \le 12\}$ defined by $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is an equivalence relation. Find the set of all elements related to 2. [5] **(b)** If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x}{1+|x|}$, show that f is one-one and onto. [5] **(c)** A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a 6. Find the probability that it is actually a 6. [5]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. **(a)** Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. [2] **(b)** Find the volume of the parallelopiped whose co-terminal edges are represented by the vectors: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the shortest distance between the lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$. [6]
2. Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost and the average cost when $x = 10$ units. Also, find the value of x for which the marginal cost equals the average cost. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ Subject to the constraints:

$$x + 3y \geq 3$$
$$x + y \geq 2$$
$$x, y \geq 0$$

   [4]

2. Given the following results for a set of data: $\bar{x} = 20$, $\bar{y} = 25$, $\sigma_x = 4$, $\sigma_y = 5$, and correlation coefficient $r = 0.8$. Find the two regression equations and estimate the value of x when $y = 30$. [6]