ISC CLASS XII MATHEMATICS (TEST PAPER 20) - SET 20

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Z}$ defined by $a * b = a + b - 1$. Find the inverse of the element 5. [1]
2. Evaluate: $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$. [1]
3. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not a bijection. [1]
4. If A is a square matrix such that $A^T = A$, what is A called? [1]
5. Find $\frac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. [1]
6. Write the value of $\int_0^a f(x)dx + \int_0^a f(a-x)dx$. [1]
7. Write the order and degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$. [1]
8. Find $\lim_{x\to 2} \frac{x^4-16}{x-2}$. [1]
9. If X is a random variable, write the formula for $\text{Var}(X)$ using $E(X)$ and $E(X^2)$. [1]
10. Let R be a relation on $\mathbb{N}$ defined by xRy if x divides y. Is R a transitive relation? Justify. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \cos(2x)$, find $\frac{d^2y}{dx^2}$. [2]
2. Use differentials to approximate the value of $\sqrt[3]{126}$. [2]
3. From 7 teachers and 4 students, a committee of 5 is to be formed. Find the probability that the committee will have exactly 3 teachers. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Find the intervals in which the function $f(x) = x^3 - 6x^2 + 5$ is strictly increasing or strictly decreasing. [4]
2. Find the particular solution of the differential equation: $\frac{dy}{dx} + y \sec x = \tan x$, given $y(0) = 1$. [4]
3. Evaluate: $\int \frac{dx}{5+4\cos x}$. [4]
4. Show that $b+c, c+a, a+b$ is a factor of the determinant $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Show that the volume of the greatest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. [6]
2. Evaluate: $\int e^{2x} \left(\frac{1-\sin 2x}{1-\cos 2x} \right) dx$. [6]
3. Solve the system of linear equations using the matrix method: [6]

$$\begin{aligned} x + y + z &= 3 \\ x - 2y + z &= 1 \\ 3x + y - 2z &= 4 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Show that the function $f : \mathbb{R} - \{\frac{3}{2}\} \to \mathbb{R} - \{\frac{1}{2}\}$ defined by $f(x) = \frac{x-2}{2x-3}$ is invertible. Find $f^{-1}(x)$. [6] **(b)** A coin is biased so that the head is 3 times as likely to occur as tails. If the coin is tossed 4 times, find the probability of getting at least 3 heads. [6] **(c)** Find the general solution of the differential equation: $\frac{dy}{dx} = \frac{x^2-y^2}{2xy}$. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the scalar component of the projection of the vector $3\hat{i} - \hat{j} + 4\hat{k}$ on the vector $2\hat{i} + 4\hat{j} - 4\hat{k}$. [2]
2. Find the value of λ for which the vectors $\hat{i} - 3\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 2\hat{k}$ and $\lambda\hat{i} + \hat{j} - \hat{k}$ are coplanar. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane that passes through the three points $(1, 1, 0)$, $(1, 2, 1)$, and $(-2, 2, -1)$. [6]
2. Using integration, find the area of the region in the first quadrant bounded by the circle $x^2 + y^2 = 9$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function is given by $C(x) = 2x + 100$. Find the marginal cost (MC) and the average cost (AC) functions. Comment on the nature of AC. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 3x + 2y$ Subject to the constraints:
$$x + 2y \leq 10$$
$$3x + y \leq 15$$
$$x \geq 0$$
$$y \geq 0$$
[4]
2. The two lines of regression are $2x - 9y + 6 = 0$ and $x - 2y + 1 = 0$. Find the means of x and y. If the standard deviation of x is 3, find the coefficient of correlation r. [6]