ISC CLASS XII MATHEMATICS (TEST PAPER 19) - SET 19

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{R}$ defined by $a * b = |a - b|$. Check if $*$ is associative. [1]
2. Evaluate: $\sin\left(\cos^{-1}\frac{3}{5}\right)$. [1]
3. State the domain of the function $f(x) = \sin^{-1}(2x^2 + 1)$. [1]
4. If $f : \{1, 2, 3\} \to \{a, b, c\}$ is given by $f(1) = a, f(2) = b, f(3) = c$, write the inverse function f^{-1}. [1]
5. Find $\frac{dy}{dx}$ if $y = \sqrt{\log(\sin x)}$. [1]
6. Write the value of $\int_0^{\pi} \cos x dx$. [1]
7. Write the order and degree of the differential equation $\frac{d^2y}{dx^2} = \left(y + \left(\frac{dy}{dx}\right)^2\right)^{1/3}$. [1]
8. Determine the value of k for which $f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 2 \\ k & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$. [1]
9. If $P(A|B) = P(A)$, what is the relationship between events A and B? [1]
10. If A is a square matrix, show that $(A^T)^T = A$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $x = at^2$ and $y = 2at$, find $\frac{d^2y}{dx^2}$. [2]

2. The radius of a cone is 4 cm and its height is 3 cm. If the height is increasing at a rate of 0.1 cm/s while the radius is kept constant, find the rate of change of the volume. [2]
3. A bag contains 4 white and 6 black balls. Two balls are drawn without replacement. Find the probability that the second ball drawn is black. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Find the equation of the tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at the point $(2, 4)$. [4]
2. Find the particular solution of the differential equation: $x\frac{dy}{dx} = y + 2\sqrt{x^2 + y^2}$, given $y(1) = 0$. [4]
3. Evaluate: $\int \frac{x}{(x+1)(x^2+1)} dx$. [4]
4. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$, solve for matrix X if $2A - 3X = B$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Show that $\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$. [6]
2. Find the point on the curve $y^2 = 8x$ which is nearest to the point $(1, 4)$. [6]
3. Evaluate: $\int \frac{dx}{3x^2+13x-10}$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that the relation R on the set $\mathbb{Z}$ of integers defined by xRy if $x - y$ is a multiple of 5, is an equivalence relation. [6] **(b)** Solve the differential equation: $\frac{dx}{dy} + \frac{x}{y} = y^2$. [4] **(c)** A factory has two machines, Machine A and Machine B. Machine A produces 60% of the items and Machine B produces 40%. 2% of the items produced by A and 1% of the items produced by B are defective. If a defective item is chosen at random, what is the probability that it was produced by Machine B? [5]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$. [2]
2. Find the area of the triangle with vertices $A(1, -1, 2)$, $B(2, 1, -1)$, and $C(3, -1, 2)$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the distance between the parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$. [6]
2. Using integration, find the area bounded by the parabola $y = x^2$ and the x-axis from $x = 0$ to $x = 3$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. A sheet of aluminum 40 cm long and 10 cm wide is to be made into a closed box with a square base. Find the dimensions of the box that will hold maximum volume. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 4y$ Subject to the constraints:
$$x + 2y \leq 10$$
$$2x + y \leq 10$$
$$x \geq 0$$
$$y \geq 0$$
[4]

2. Given $b_{xy} = 0.5$, $r = 0.7$, and the standard deviation of y is $\sigma_y = 4$. Find the standard deviation of x, σ_x. [6]