ISC CLASS XII MATHEMATICS (TEST PAPER 4) - SET 04

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Q}$ defined by $a * b = a + b - ab$. Find the identity element for $*$. [1]
2. Simplify: $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, where $\frac{a}{b}\tan x > -1$. [1]
3. Determine if the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(\pi x)$ is one-one. [1]
4. Find the point of discontinuity for the function $f(x) = \frac{x^2-4x+3}{x^2-1}$. [1]
5. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(x\sqrt{1-x^2}\right)$. [1]
6. Evaluate: $\int \frac{1}{\sqrt{x^2-8x+15}}dx$. [1]
7. Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} - 3y = \sin x$. [1]
8. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.5$, find $P(A \cap B')$. [1]
9. A discrete random variable X has mean $E(X) = 4$. Find the mean of $Y = 3X + 5$. [1]
10. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, find $|A^3|$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. Verify Rolle's Theorem for the function $f(x) = e^x \sin x$ on the interval $[0, \pi]$. [2]
2. Differentiate $y = (\log x)^x + x^{\log x}$ with respect to x. [2]
3. The probability of solving a problem by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Find the probability that the problem is solved. [2]

Question 3 (4×4 Marks $= 16$ Marks)

Answer the following questions.

1. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$. [4]
2. A cube's edge is increasing at the rate of 3 cm/s. How fast is its volume increasing when the length of the edge is 10 cm? [4]
3. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} dx$. [4]
4. Find the values of x and y such that the matrix $A = \begin{pmatrix} 0 & 2y & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{pmatrix}$ is skew-symmetric. [4]

Question 4 (3×6 Marks $= 18$ Marks)

Answer the following questions.

1. Show that the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $2ab$. [6]
2. Evaluate: $\int e^{2x} \sin(3x) dx$. [6]
3. Solve the linear differential equation: $(1 + y^2)dx + (x - e^{\tan^{-1} y})dy = 0$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 + 4$. Show that f is not invertible. If the domain is restricted to $[0, \infty)$, show f is invertible and find its inverse. [6] **(b)** Solve the system of linear equations using the matrix method: [6]

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

   (c) A factory has three machines A, B, and C. Machine A produces 50% of the total output, B produces 30%, and C produces 20%. The defective percentage from A, B, and C are $2\%, 3\%$, and 5% respectively. If an item is drawn at random and is found to be defective, find the probability that it was produced by Machine A. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the area of the parallelogram whose adjacent sides are the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. [2]
2. If the vectors $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 4\hat{k}$, and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$ form the sides of a triangle, find the length of the altitude from vertex A to the side represented by $\vec{b}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the two planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$. [6]
2. Using integration, find the area of the region bounded by the line $y = x$ and the parabola $y = 2x - x^2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The revenue function is given by $R(x) = 15x - x^2$ and the cost function is $C(x) = 5x + 7$. Find the output level x at which the marginal revenue equals the marginal cost. Determine the profit at this output level and find the price p when the marginal profit is zero. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 5x + 3y$ Subject to the constraints:
$$3x + 5y \leq 15$$
$$5x + 2y \leq 10$$
$$x, y \geq 0$$
[4]

2. A study yielded the following information: Mean of $x = 20$, Mean of $y = 15$, Standard deviation of $x = 4$, Standard deviation of $y = 3$, Regression coefficient $b_{yx} = 0.7$. Find the two regression equations and the coefficient of correlation (r). [6]