ISC CLASS XII MATHEMATICS (TEST PAPER 3) - SET 03

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Find the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. [1]
2. Find the domain of the function $f(x) = \sin^{-1}(2x-3)$. [1]
3. If $x^2+y^2=1$, find $\frac{dy}{dx}$ in terms of y. [1]
4. If $y = e^{\tan x}$, find $\frac{dy}{dx}$. [1]
5. Find the value of $\int_0^2 |x-1|dx$. [1]
6. Write the integrating factor (I.F.) of the differential equation: $\frac{dy}{dx} - y\tan x = \sec x$. [1]
7. If $P(A') = 0.6$ and $P(B') = 0.5$, and $P(A\cap B) = 0.2$, find $P(A\cup B)$. [1]
8. If $P(A) = \frac{3}{5}$ and $P(B|A) = \frac{1}{3}$, find $P(A\cap B)$. [1]
9. A particle moves along the curve $y = \frac{2}{3}x^3+1$. Find the points on the curve where the y-coordinate is changing twice as fast as the x-coordinate. [1]
10. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$, find the matrix $A^2 - 2A - 7I$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \tan^{-1} x$, show that $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$. [2]
2. Use differentiation to approximate the value of $\sqrt{49.5}$. [2]

3. A random variable X has the following probability distribution:

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Find the mean $E(X)$. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Prove that the function $f(x) = 2x + \cos x$ is strictly increasing on $\mathbb{R}$. [4]
2. Solve the differential equation: $(x^2 - y^2)dx + 2xydy = 0$. [4]
3. Evaluate: $\int \frac{x}{(x-1)^2(x+2)} dx$. [4]
4. Without expanding the determinant, prove that:

$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

[4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. A window is in the form of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 metres, find the dimensions so that the greatest possible light is admitted (i.e., the area is maximum). [6]
2. Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$. [6]
3. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{pmatrix}$. Find A^{-1} using the Adjoint method. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** If $f(x) = 4x + 3$ and $g(x) = \frac{x-3}{4}$, verify that $f \circ g = I_D$ and $g \circ f = I_R$, and hence write down the inverse function f^{-1}. [6] **(b)** An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a car driver? [6] **(c)** Prove that $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the point $(1, 2, 1)$ and perpendicular to the line joining the points $(1, 4, 2)$ and $(2, 3, 5)$. [2]
2. Find the area of the triangle formed by the points $A(1, 1, 1)$, $B(1, 2, 3)$, and $C(2, 3, 1)$ using vector methods. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the vector and cartesian equations of the line passing through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. Hence, find the distance of the point $(0, 0, 0)$ from this line. [6]
2. Using integration, find the area of the region bounded by the curves $y = 6x - x^2$ and $y = x^2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The demand function for a certain commodity is $p = 15 - 2x^2$ and the cost function is $C(x) = x^3 + 2x - 1$. Find the profit function $P(x)$. Find the level of output x at which the profit is maximum. [5]

Question 9 (10 Marks)

Answer the following questions.

1. A fruit grower has 1500 large apples and 4000 small apples. He can sell two assortments: Assortment A (1 large, 2 small, profit Rs. 5) and Assortment B (2 large, 1 small, profit Rs. 6). Formulate this as an LPP and solve graphically to find the maximum profit. [4]

2. Calculate the two regression coefficients and use them to find the coefficient of correlation (r) from the following data:

x	10	12	13	14	16
y	15	18	20	21	26

[6]