ISC CLASS XII MATHEMATICS (TEST PAPER 10) - SET 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 $\times$ 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Q}$ defined by $a * b = a + b - 1$. Find the identity element for $*$. [1]
2. Find the value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$. [1]
3. State the range of the function $f(x) = \sin^{-1}(2x)$. [1]
4. Determine if the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 5$ is one-one. [1]
5. Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{x})$. [1]
6. Write the general solution of the differential equation $\frac{dy}{dx} = 4x^3$. [1]
7. Find the value of k such that $f(x) = \begin{cases} \cos x & \text{if } x \leq 0 \\ kx^2 + 1 & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$. [1]
8. Evaluate: $\int \frac{1}{\sqrt{4-9x^2}} dx$. [1]
9. If $P(A) = 0.3$, $P(B) = 0.4$, and $P(A \cup B) = 0.6$. Find $P(A' \cap B')$. [1]
10. Write the formula for the variance of a Binomial distribution $B(n, p)$. [1]

Question 2 (3 $\times$ 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \cos(m \cos^{-1} x)$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$. [2]
2. A man 1.8 m tall walks away from a lamp post 5 m high at the rate of 1.2 m/s. Find the rate at which the length of his shadow is increasing. [2]
3. A box contains 10 items, 3 of which are defective. A sample of 2 items is drawn from the box. Find the probability that the sample contains exactly 1 defective item. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Using matrix methods, find the cofactors C_{21} and C_{33} for $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 3 & 2 & 5 \end{pmatrix}$. Hence, find the value of $|A|$. [4]
2. Evaluate: $\int x \sec^2 x dx$. [4]
3. Find a point on the curve $f(x) = x^2 + 2x - 8$ in the interval $[-4, 2]$ where the tangent is parallel to the chord joining the end points. [4]
4. Solve the differential equation: $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Evaluate: $\int \frac{1}{\cos^4 x+\sin^4 x} dx$. [6]
2. Show that for a given perimeter, the area of a trapezoid with non-parallel sides equal is maximum when it is an equilateral trapezoid. [6]
3. Prove that: $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)^2$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that the relation R on the set of 2×2 matrices with real entries, defined by ARB if $\det(A) = \det(B)$, is an equivalence relation. [6] **(b)** A random variable X has the following probability distribution:

X	0	1	2	3	4
$P(X)$	0.1	0.2	0.3	0.3	0.1

Find the mean $E(X)$ and the variance $\mathrm{Var}(X)$. [6] **(c)** Prove that A and B are independent events if and only if A and B' are independent. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the scalar triple product $[\vec{a} \cdot (\vec{b} \times \vec{c})]$ if $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$. [2]
2. Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$. [6]
2. Using integration, find the area bounded by the parabola $x^2 = y$ and the line $y = x + 2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost $C(x)$ and the revenue $R(x)$ from the production and sale of x units of a product are given by $C(x) = 5x + 350$ and $R(x) = 50x - 2x^2$. Find the value of x that maximizes the profit. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = x + 2y$ Subject to the constraints:

$$\begin{aligned} 2x + y &\geq 3 \\ x + 2y &\geq 6 \\ x, y &\geq 0 \end{aligned}$$

[4]

2. Given the correlation coefficient $r = 0.8$, standard deviations $\sigma_x = 3$ and $\sigma_y = 5$, and means $\bar{x} = 10$ and $\bar{y} = 20$. Find the regression equation of y on x and estimate the value of y when $x = 13$. [6]